UNIVERSIDADE DE SÃO PAULO
ESCOLA DE ENGENHARIA DE SÃO CARLOS
DEPARTAMENTO DE ENGENHARIA DE ESTRUTURAS

DISTRIBUIÇÃO DAS FORÇAS DEVIDAS AO VENTO ENTRE OS PAINÉIS DE CONTRAVENTAMENTO

ÉDDIE MANCINI
WALTER SAVASSI
MARIA ÂNGELA PEREIRA XAVIER

DIGITAÇÃO DO TEXTO:
Rui Roberto Casale
Antonio Valdair Carneiro

ELABORAÇÃO DOS DESENHOS:
Francisco Carlos Guete de Brito
Sylvia Helena Morette

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INTRODUÇÃO

Estruturas usuais de edifícios altos são compostas de lajes, pilares, pilares-parede, vigas, núcleos estruturais que têm por finalidade conduzir as forças verticais de utilização dos mesmos até às fundações.

Os elementos estruturais projetados para receber as cargas verticais são também aptos para resistir às forças horizontais provenientes da ação do vento. As estruturas planas formadas por pilares, pilares-parede, vigas, constituem o que chamamos painéis de contraventamento.

Admitimos então que as forças horizontais sejam aplicadas às lajes e estas as distribuem entre estes painéis.

A questão teórica da determinação das forças recebidas por estes painéis tem sido exaustivamente estudada. Trabalhos pioneiros foram realizados no Departamento de Engenharia de Estruturas da Escola de Engenharia de São Carlos pelo saudoso Prof. Miguel Carlos Stamato [1], [2]. Entre outros podemos citar os trabalhos de MANCINI [3], [4], COELHO [5], sendo que alguns destes textos enfocavam o assunto utilizando técnicas contínuas de análise em vez das consagradas técnicas discretas.

Quanto aos trabalhos que utilizam a chamada técnica do meio contínuo, dado o estágio inicial de desenvolvimento desta técnica, algumas simplificações eram feitas para viabilizar o tratamento matemático dos problemas. Assim, normalmente eram desconsideradas as deformações axiais dos pilares nas formulações teóricas. Tal hipótese, que visava viabilizar o tratamento via técnica contínua, tornava restritiva a aplicação das rotinas de cálculo desenvolvidas. Para edifícios de maior altura e em outros casos particulares (vigas de grande altura ligando os pilares-parede) o erro na avaliação da rigidez de cada painel resistente e portanto na distribuição da força horizontal entre os painéis era considerável, o que indicava a utilização da técnica contínua para cálculos de ante-projeto.

Entretanto, durante os últimos anos diversas pesquisas foram realizadas na EESC-USP visando incluir nas análises a consideração das deformações axiais. Entre os trabalhos que tiveram êxito nesta finalidade podemos citar BATTISTELLE [6], YANEZ [7], XAVIER [8], MANCINI E SAVASSI [9], MANCINI [10]. O presente trabalho também pretende resolver diversos problemas superando aquela restritiva hipótese simplificadora.

Em alguns casos os cálculos ainda continuam aproximados mas sem a simplificação grosseira de pilares rígidos axialmente.

Quanto à técnica do meio contínuo em si, esta consiste na substituição de um conjunto discreto de peças estruturais (vigas, cabos, lajes, etc.) por um "meio contínuo" de rigidez equivalente. No caso de estruturas de edifícios altos, as lajes dos andares, em número finito, são substituídas por uma infinidade de diafragmas horizontais ligando continuamente os diversos painéis de contraventamento. Tal substituição altera a natureza matemática da solução dos diversos problemas. As incógnitas, em número finito, transformam-se em funções incógnitas analíticas e, em geral, a solução de sistemas de equações lineares é substituída pela solução de equações ou sistemas de equações diferenciais.

Tal hipótese, caso haja um número suficiente de elementos discretos, pouco altera os resultados procurados nos diversos problemas que são, em geral, deslocamentos e esforços. Muitas vezes, na utilização da idéia do meio contínuo em estruturas de edifícios altos, adicionam-se, para a simplificação dos cálculos, hipóteses adicionais (p. ex. momento fletor nulo nos pontos médios de vigas e pilares de pórtico; consideração de pilares indeformáveis axialmente, etc.). Tais hipóteses entretanto não fazem parte da concepção fundamental da técnica do meio contínuo.

Adiantamos que a técnica contínua leva a resultados praticamente coincidentes com outras técnicas de cálculo, desde que não se introduzam hipóteses adicionais conforme referido acima.

A aplicação da técnica do meio contínuo é vantajosa em estruturas regulares em planta e elevação, não se restringindo, entretanto, a sua aplicação a casos em que não haja variação das características geométricas.

No caso de estruturas de edifícios altos, tem-se usado, com êxito, esta técnica, em estruturas com diversos trechos de rigidez uniforme e variáveis entre si.

Outra possibilidade muito explorada tem sido a formulação adimensional dos problemas, que é bastante facilitada quando se trabalha com a técnica contínua.

A técnica do meio contínuo pode também ser acoplada ao método dos elementos finitos conforme procedimento idealizado de forma inédita por SAVASSI [11]. É possível agora ser superada a restritiva condição de estrutura uniforme na aplicação da técnica, a par da utilização de processo numérico de eficiência comprovada na solução das equações diferenciais.

Finalizando, queremos crer, a partir da experiência acumulada em três décadas de pesquisa com a técnica contínua que esta possui potencialidade para tornar-se ferramenta corrente na análise de estruturas de edifícios altos conforme pode se comprovar pelos trabalhos de YANEZ [7], XAVIER [8], FAKURY [12], [13], e outros.

Lembramos que outra característica importantíssima desta técnica é a possibilidade de se estudar o comportamento global das estruturas, como um todo, em conseqüência do pequeno número de parâmetros requeridos nas análises.

Incentivamos assim os pesquisadores a trabalhar para que toda esta potencialidade possa ser explorada e atualizada, fornecendo assim mais um recurso valioso para o engenheiro estrutural, em particular o projetista de estruturas de edifícios altos.

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THE STRUCTURAL DESIGN OF TALL BUILDINGS
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TALL BUILDING STRUCTURES UNIFIED PLANE PANELS BEHAVIOUR

EDDIE MANCINI AND WALTER SAVASSI*
Department of Structural Engineering, São Carlos School of Engineering-USP, Caixa Postal 359, 13560-970 São Carlos SP, Brazil

SUMMARY

In this paper, it is shown that every plane panel, used to brace tall building structures, can be easily and generally approached through the use of the continuous medium technique. As is well known, the number of parameters needed to define any panel is very small, both in relation to its stiffness or behaviour characteristics, when acted upon by lateral wind forces. Initially, by deriving the governing differential equations of the panel behaviour, the equivalence is shown of the formal mathematical description, and hence of the structural behaviour, between a pair of shear walls associated by lintel beams and the plane association, by pinned horizontal bars, of one shear wall and one frame. In both cases, axial deformations due to axial forces on vertical members are taken into account. Copyright © 1999 John Wiley & Sons, Ltd.

1. SERIAL SHEAR WALL ASSOCIATION

The differential equation for the horizontal displacements of a serial shear wall association as shown in Figure 1 is derived below.

The continuous medium technique assumes that the shear forces acting on the horizontal connecting beams are replaced by an equivalent distributed force through the vertical z-coordinate, M. Albiges and J. Goulet[1] (1960). For convenience, in the foregoing preliminary local approach (looking for the differential equation), the structure is assumed to have uniform geometry and loading, and to be rigidly built-in at the base. If a global approach were to be used, with an approximated one-dimensional finite element panel discretization, together with the equivalent continuous medium solution, those characteristics would be free to vary.

Consider the panel deformation as shown in Figure 2.

There one can see that

$$f_t = f_f + f_n' + f_n'' = f_f + f_n \tag{1}$$

where f_f is the final distance between the considered two shear wall cross sections, after deformation, and f_n is the sum of their axial deformations due to axial forces; G_1 and G_2 are the cross sections geometric centres and $(2a)$ is the distance between the points of effective built-in connection of a lintel beam into the respective wall; the position of such points can be determined by the analysis of the elastic stress distribution in the region of beam-wall intersection.

* Correspondence to: W. Savassi, Department of Structural Engineering, São Carlos School of Engineering-USP, Caixa Postal 359, 13560-970 São Carlos SP, Brazil
E-mail: savassi@sc.usp.br

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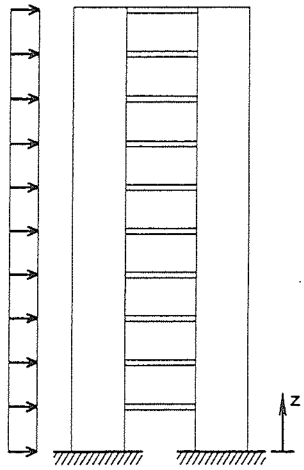

Figure 1. Shear walls connected by lintel beams

Defining $u(z)$ to be the horizontal deflection of the shear wall association:

$$f_t = (b_1 + 2a + b_2)u' = 2cu' \tag{2}$$

If V_ℓ is the generic horizontal beam shear force, i is its second moment of area and E is the material longitudinal modulus of elasticity, then

$$f_f = \frac{2V_\ell a^3}{3Ei} \tag{3}$$

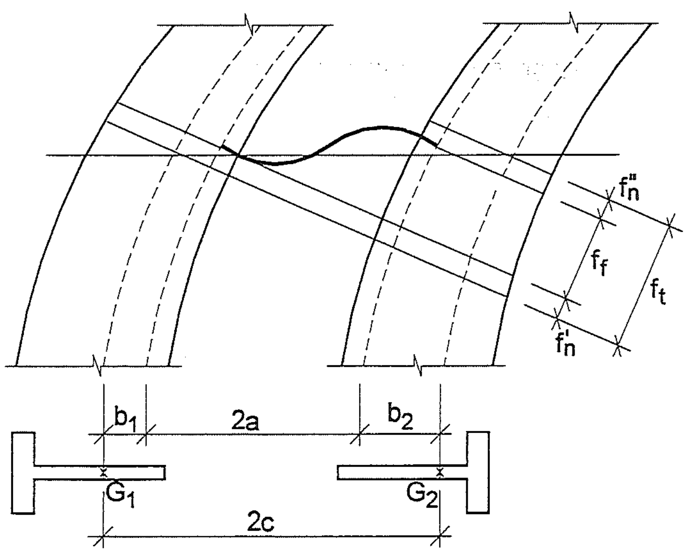

Figure 2. Panel deformations

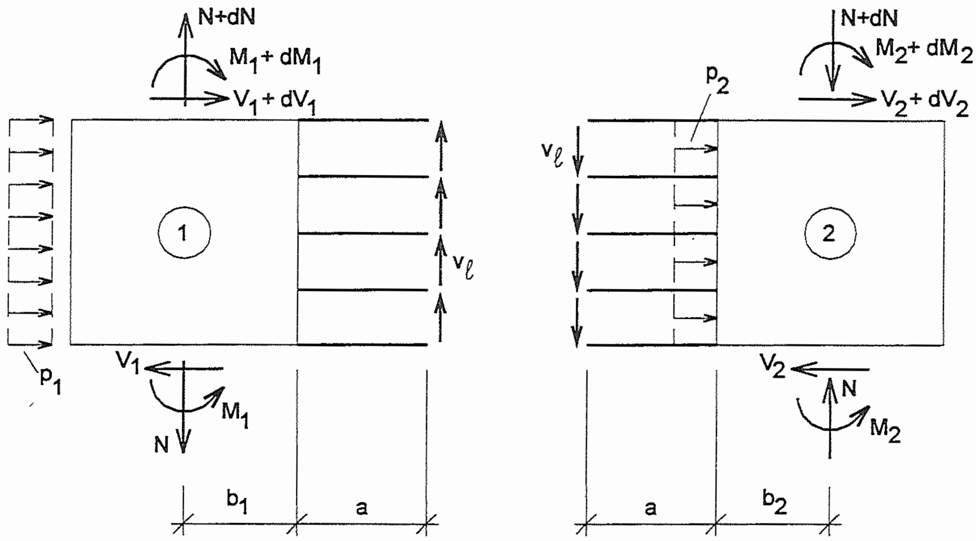

Figure 3. Shear wall infinitesimal elements

If A_1 and A_2 are the cross sectional areas of vertical elements and N the axial force, one finds

$$f_n = \frac{1}{E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_0^z N\mathrm{d}z \tag{4}$$

From equations (1), (2), (3) and (4) one obtains

$$2cu' = 2\frac{V_\ell a^3}{3Ei} + \frac{1}{E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_0^z N\mathrm{d}z \tag{5}$$

which is the compatibility equation for the displacements.

Let us now consider the shear wall infinitesimal elements ($\mathrm{d}z$) and the corresponding internal forces.

As shown in Figure 3, it is seen that, by hypothesis, the mid-points of lintel beam lengths are assumed to have zero bending moment.

Assuming for the deflected curve of the walls (shear walls shear deformations not considered), the relation

$$u'' = \frac{M}{EI_j} \tag{6}$$

by imposing rotational equilibrium of shear wall 1 infinitesimal element, with second moment of inertia I_1 and shear force V_1, one may write the equation

$$V_1 = -EI_1 u''' + \frac{V_\ell}{h}(a + b_1) \tag{7}$$

where h is the constant height between floors.

Similarly for wall 2:

$$V_2 = -EI_2 u''' + \frac{V_\ell (a + b_2)}{h} \tag{8}$$

and then, by summation, it follows that

$$V = -E(I_1 + I_2)u''' + (2c)\frac{V_\ell}{h} \tag{9}$$

where V is the shear force due to the external loading.

By imposing vertical translation equilibrium to infinitesimal elements of walls 1 or 2, we obtain

$$\frac{\mathrm{d}N}{\mathrm{d}z} = -\frac{V_\ell}{h} = -v_\ell \tag{10}$$

Taking equations (2), (5), (9) and (10) into account, one can find that

$$\alpha(j_1 + j_2)u^{\mathrm{v}} - j_{\mathrm{c}} u''' = V - \alpha V'' \tag{11}$$

where j_1 and j_2 are the bending rigidity moduli EI_1 and EI_2 of shear walls 1 and 2, respectively.

In equation (11), α and j_{c} take the following values, taking

$$M_{\mathrm{s}} = \frac{2c}{\left(\frac{1}{A_1} + \frac{1}{A_2}\right)} \tag{12}$$

i.e.

$$\alpha = \frac{hM_{\mathrm{s}}(2a)^3 E}{24cj_\ell} \tag{13·1}$$

and

$$j_{\mathrm{c}} = j_1 + j_2 + (2c)M_{\mathrm{s}}E \tag{13·2}$$

where j_ℓ is the rigidity bending modulus Ei of the generic lintel beam.

The boundary conditions for the fifth-order differential equation are, initially:

(a) at the base

$$u(0) = 0 \tag{14·1}$$

$$u'(0) = 0 \text{ (rigidly built-in)} \tag{14·2}$$

(b) at the top

$$u''(H) = 0 \text{ (no bending moments applied at the top of the shear walls)} \tag{14·3}$$

From the compatibility equation (5) one can immediately conclude (for $u'(0) = 0$)

$$V_\ell(0) = 0 \tag{15}$$

Differentiating once all the terms of that equation and knowing that $u''(H) = 0$ and $N(H) = 0$, results in

$$V'_\ell(H) = 0 \tag{16}$$

Using the global equilibrium equation (9), evaluated at the base and at the top, and taking into account equations (15) and (16), one finds, respectively:

$$u'''(0) = -\frac{V(0)}{E(I_1 + I_2)} \tag{17}$$

and

$$u^{\mathrm{IV}}(H) = -\frac{V'(H)}{E(I_1 + I_2)} \tag{18}$$

which are the two remaining boundary conditions for equation (11).

2. PLANE FRAME

Before proceeding to analyse the plane frame shear wall association, a twin columns plane frame analysis, under the action of lateral loading, and considering the influence of their axial deformations, is presented.

Figure 4 shows the deformed shape of one frame storey, one part of it due to the storey distortion and another one due to the columns A and B axial deformations.

Axial column displacements are defined as w_1 and w_2, k_j is the column j rigidity ($j = 1, 2$) calculated by the quotient I_i/h; k_v is the beam rigidity $(k_v = I_v/\ell_v)$ β is the frame node rotation and u' is the rotation of the chord of the columns deflected line. Letter I always refers to the second moments of area.

In the case where columns 1 and 2 have equal rigidities ($k_1 = k_2 = k_p$) and following the Grinter signal rule (positive counter-clockwise node rotations) the bending moments applied by the bars on node A are:

$$M_{AC} = 6Ek_p(u' - \beta) \tag{19}$$

$$M_{AE} = 6Ek_p(u' - \beta) \tag{20}$$

$$M_{AB} = -6Ek_v\left[\beta - \frac{(w_1 - w_2)}{\ell_v}\right] \tag{21}$$

The rotational equilibrium of node A will give

$$\beta = \frac{2k_p u' + k_v(w_1 - w_2)/\ell_v}{(2k_p + k_v)} \tag{22}$$

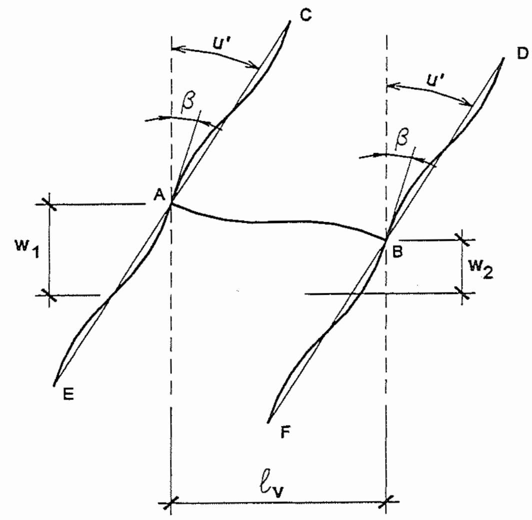

Figure 4. Deformation of a generic frame storey

Assuming that the columns mid-points (between adjacent stories) have zero bending moments, the frame shear force V_f can be written as

$$V_f = \frac{2}{h}(M_{AC} + M_{BD}) \tag{23}$$

Using equation (19), and knowing that $M_{BD} = M_{AC}$, results in

$$V_f = \frac{24Ek_p}{h}(u' - \beta) \tag{24}$$

Substituting equation (22) into equation (24), one finds

$$V_f = s_f u' - r_1(w_1 - w_2) \tag{25}$$

where

$$s_f = \frac{24Ek_p k_v}{h(2k_p + k_v)} \tag{26·1}$$

$$r_1 = \frac{s_f}{\ell_v} \tag{26·2}$$

Figure 5 shows the generic beam internal actions (assumed positive), where V_ℓ is the beam shear force, concentrated at the storey level, and v_ℓ is the vertically distributed shear force, through the columns height, as normally assumed as one of the basic hypotheses of the continuous medium technique.

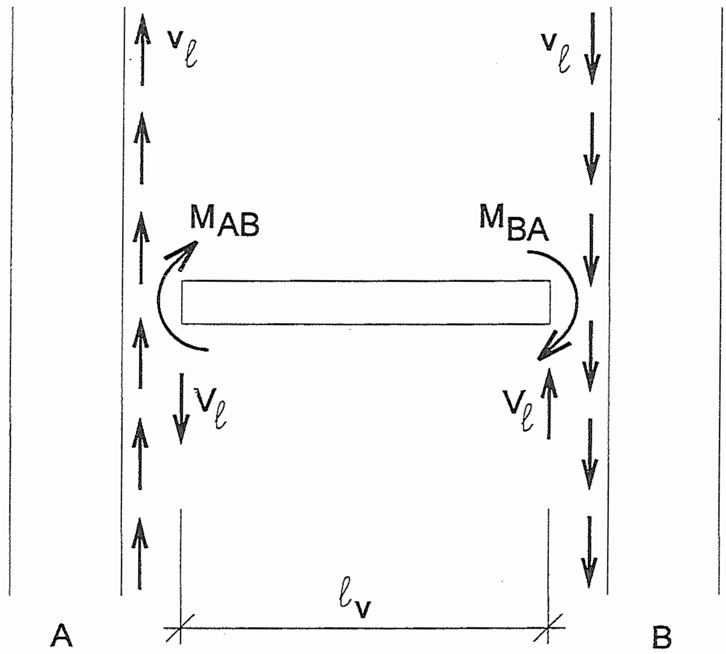

Figure 5. Internal actions on beam AB

According to Figure 5, one has

$$v_\ell = \frac{1}{h\ell_v}(M_{AB} + M_{BA}) \tag{27}$$

and hence, from equation (21),

$$v_\ell = c_1 u' - d_1(w_1 - w_2) \tag{28}$$

where

$$c_1 = \frac{24Ek_p k_v}{h(2k_p + k_v)} = \frac{s_f}{\ell_v} \tag{29·1}$$

$$d_1 = \frac{c_1}{\ell_v} = \frac{s_f}{\ell_v^2} \tag{29·2}$$

In Figure 6 a generic infinitesimal element of column 1 is presented with its respective loading v_ℓ and internal actions.

Assuming the directions shown in Figure 6 to be positive, from the translation vertical equilibrium of column 1 generic element will result ($N_A \equiv N_1$)

$$\frac{dN_1}{dz} = -v_\ell \tag{30}$$

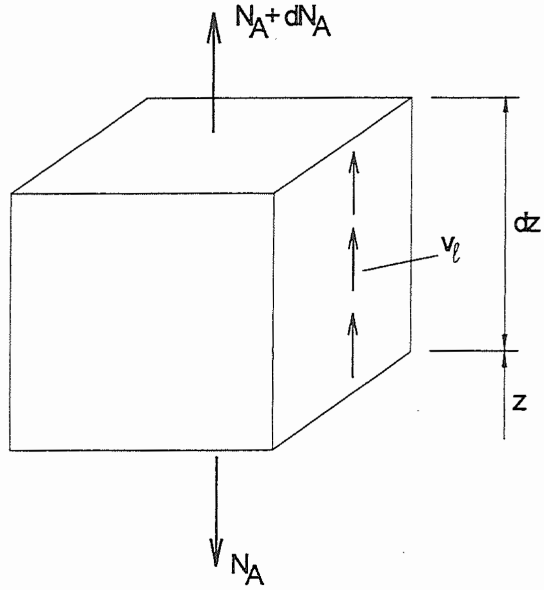

Figure 6. Column 1 generic infinitesimal element

Since the column 1 axial force is given by

$$N_1 = EA_p w'_1 \tag{31}$$

where A_p is the column cross sectional area, combining equations (30) and (31) will give

$$EA_p w''_1 = -v_\ell \tag{32}$$

Similarly, from the equilibrium of the column 2 generic element one gets

$$EA_p w''_2 = v_\ell \tag{33}$$

Extracting now the u' unknown from equation (25) and substituting the value of v_ℓ from equation (28) into equations (32) and (33), will give

$$EA_p w''_2 = -EA_p w''_1 = \frac{V_f}{\ell_v} \tag{34}$$

because

$$\frac{c_1}{s_f} = \frac{1}{\ell_v} \quad \text{and} \quad \frac{d_1}{r_1} = \frac{1}{\ell_v} \tag{35}$$

The following boundary conditions should be used to solve the differential equations:

(a) at the base ($z = 0$)

$$u(0) = 0 \tag{36·1}$$

$$w_1(0) = 0 \quad \text{and} \quad w_2(0) = 0 \tag{36·2}$$

(b) at the top ($z = H$)

$$w'_1(H) = 0 \quad \text{for } N_1(H) = 0 \tag{37·1}$$

$$w'_2(H) = 0 \quad \text{for } N_2(H) = 0 \tag{37·2}$$

If the external loading is assumed to be a uniformly distributed horizontal force p through the height of the frame, then

$$V_f = p(H - z) \tag{38}$$

Substituting equation (38) into equation (34), integrating, and using boundary conditions given by equations (36·1, 36·2), (37·1 and 37·2), will result in

$$w_2 = -w_1 = \frac{pz}{6EA_p\ell_v}(-z^2 + 3Hz - 3H^2) \tag{39}$$

Substituting the expressions for w_1 and w_2 from equation (39) into equation (25) and taking equation (38) into account will result in a u' expression which, once integrated, using the boundary condition given by equation (36·1), allows us to obtain the frame horizontal displacement expression:

$$u_f = pz\left[\frac{(2H - z)}{2s_f} + z\frac{(z^2 - 4Hz + 6H^2)}{12EA_p\ell_v^2}\right] \tag{40}$$

This equation is made of two parts, the first one due to the frame deformation by shear forces and the second one due to columns axial deformations (and hence due to the frame deformation by the bending moment). This equation can then be written in the form

$$u_f = u_{fv} + u_{fm} \tag{41}$$

and it is possible to show that if V_f and M_f are, respectively, the shear force and the bending moment due to external loading, then

$$u'_{fv} = \frac{p(H - z)}{s_f} = \frac{V_f}{s_f} \tag{42}$$

and

$$u''_{\rm fm} = \frac{p(H-z)^2}{EA_p\ell_v^2} = \frac{M_f}{j_f} \tag{43}$$

where s_f is the frame shear rigidity, j_f is the frame modulus of rigidity (EI_f), and I_f is the total second moment of area of the columns cross sections referred to the their common geometric centre, but not considering their own second moment of area in relation to their individual geometric centres. E is the material Youngs modulus.

The preceding results may be extended to frames with unequal columns, and in that case

$$I_f = \frac{\ell_v^2}{\left(\frac{1}{A_1} + \frac{1}{A_2}\right)} \tag{44}$$

where A_1 and A_2 are the columns cross sectional areas and

$$s_f = \frac{12E}{h}\sum_{\rm ns}\left[k_{\rm c.n}\frac{\sum_{\rm b.n} k}{\sum_{\rm m.n} k}\right] \tag{45}$$

where

- k is the relation I/l of the (members) beam or column under consideration
- $\sum_{\rm n.s}$ is the summation extended to every *n*ode of the considered *s*torey
- $k_{\rm c.n}$ is the relation I/l of the *c*olumn above the considered *n*ode
- $\sum_{\rm m.n} k$ is the summation extended to every concurrent *m*ember at the *n*ode
- $\sum_{\rm b.n} k$ is the summation extended to every concurrent *b*eam at the *n*ode
- E is the material Youngs modulus
- h is the storey height.

3. SHEAR WALL

A shear wall is a plane panel that is deformable by bending moment action and extremely rigid to shear force action, hence having as its governing equation

$$V_w = -j_w u''' \tag{46}$$

where j_w (w standing for wall) is the bending rigidity modulus EI of the wall and V_w the shear force.

The assumed positive directions for internal actions are shown in Figure 7.

4. SHEAR WALL–FRAME ASSOCIATION BY PINNED BARS

Many building structures acted upon by horizontal forces can be analysed using the model shown in Figure 8, which represents a shear wall and a frame linked together by horizontal pinned bars, which are assumed to have no axial deformation. Those bars take the role of floor slabs, assumed to be rigid to membrane (horizontal) action.

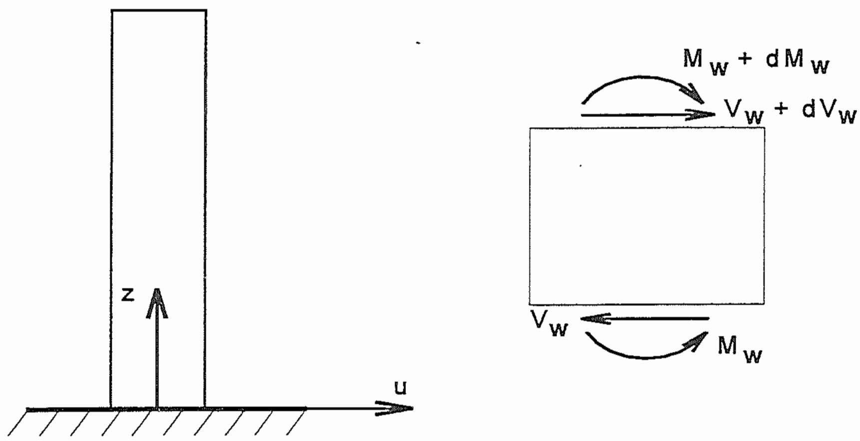

Figure 7. Shear wall

In the continuous medium technique, the horizontal connecting pinned bars are assumed to be continuously distributed through the panel height.

As already shown, the frame deflected line equation, considering columns axial deformations, is given by equation (41).

Considering equations (42) and (43), one may write, Battistelle[2] (1991)

$$u_f = \int_0^z \frac{V_f}{s_f} \mathrm{d}z + \int_0^z \int_0^z \frac{M_f}{j_f} \mathrm{d}z^2 \tag{47}$$

But, as is known, in this structure

$$V = V_w + V_f \tag{48}$$

and

$$M = M_w + M_f \tag{49}$$

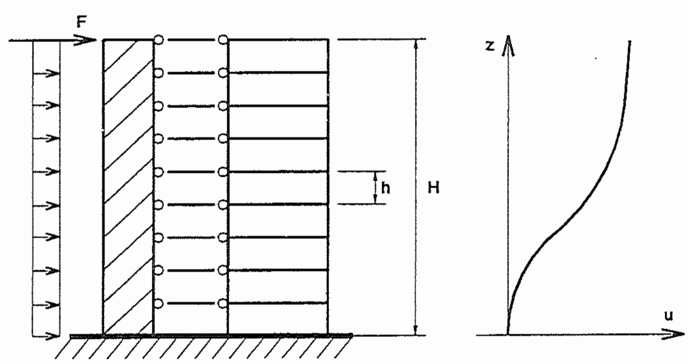

Figure 8. Shear wall–frame association by pinned bars

where
V is the panel shear force, at level z from the base, due to external loading
M is the panel bending moment, at level z, due to the external loading.

The shear wall deflected line equation, neglecting shear force deformation, is given by

$$u''_{\mathrm{w}} = \frac{M_{\mathrm{w}}}{j_{\mathrm{w}}} \tag{50}$$

and substituting equation (50) and its first derivative into (48) and (49) will give

$$V_{\mathrm{f}} = V + j_{\mathrm{w}} u'''_{\mathrm{w}} \tag{51}$$

and

$$M_{\mathrm{f}} = M - j_{\mathrm{w}} u''_{\mathrm{w}} \tag{52}$$

For a continuous connection $u_{\mathrm{w}} = u_{\mathrm{f}} = u$ and, therefore, substituting equations (51) and (52) into equation (47), one may write

$$u = \int_0^z \frac{(V + j_{\mathrm{w}} u''')}{s_{\mathrm{f}}} \mathrm{d}z + \int_0^z \int_0^z \frac{(M - j_{\mathrm{w}} u'')}{j_{\mathrm{f}}} \mathrm{d}z^2 \tag{53}$$

Differentiating this equation twice, one obtains

$$\frac{j_{\mathrm{w}}}{s_{\mathrm{f}}} u^{\mathrm{IV}} - \left(1 + \frac{j_{\mathrm{w}}}{j_{\mathrm{f}}}\right) u'' = -\frac{V'}{s_{\mathrm{f}}} - \frac{M}{j_{\mathrm{f}}} \tag{54}$$

The boundary conditions for this equation are

(a) at the base ($z = 0$)

$$u(0) = 0 \tag{55·1}$$

$$u'(0) = 0 \tag{55·2}$$

(b) at the top ($z = H$)

$$u''(H) = 0 \tag{56·1}$$

$$V_{\mathrm{w}}(H) + V_{\mathrm{f}}(H) = V(H) \tag{56·2}$$

To show the explicit form of equation (56·2), one recalls that values of V_{w} and V_{f} are given, respectively, by equations (46) and (25), repeated in the following, for $z = H$.

$$V_{\mathrm{w}}(H) = -j_{\mathrm{w}} u'''(H) \tag{57}$$

and

$$V_{\mathrm{f}}(H) = s_{\mathrm{f}} u'(H) - r_1[w_1(H) - w_2(H)] \tag{58}$$

where

$$s_{\mathrm{f}} = r_1 \ell_{\mathrm{v}} = \frac{24Ek_p k_{\mathrm{v}}}{h(2k_p + k_{\mathrm{v}})} \tag{59}$$

and

$$[w_1(H) - w_2(H)] = \frac{1}{E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right) \int_0^H \frac{(M - j_{\mathrm{w}} u'')}{\ell_{\mathrm{v}}} \mathrm{d}z \tag{60}$$

Putting equations (57) and (60) into equation (56·2) results in

$$V(H) = -j_{\mathrm{w}} u'''(H) + s_{\mathrm{f}} u'(H) - \frac{s_{\mathrm{f}}}{E\ell_{\mathrm{v}}^2}\left(\frac{1}{A_1} + \frac{1}{A_2}\right) \int_0^H (M - j_{\mathrm{w}} u'')\mathrm{d}z \tag{61}$$

Before going into the formal mathematical equivalence between the behaviour of both shear walls associated by lintel beams and frame–shear walls associated by pinned bars, to help in that aspect, one derives the governing differential equation for the first of these two panels, when no column axial deformation is taken into account. In this case, the displacement compatibility equation (5) will be, with $A_1 \to \infty$ and $A_2 \to \infty$,

$$2cu' = \frac{2V_\ell a^3}{3Ei} \tag{62}$$

from which one can find V_ℓ, and, taking it to equation (9), get

$$V = -E(I_1 + I_2)u''' + \frac{3Ei}{2h}\frac{(2c)^2}{a^3}u' \tag{63}$$

or

$$V = -j_{\mathrm{g}} u''' + s_{\mathrm{g}} u' \tag{64}$$

where

$$j_{\mathrm{g}} = -(j_1 + j_2) = -E(I_1 + I_2) \tag{65}$$

$$s_{\mathrm{g}} = \frac{3Ei}{2h}\frac{(2c)^2}{a^3} \tag{66}$$

If, in the frame equation (25), axial displacements w_1 and w_2 were not considered, i.e. $w_1 = 0$ and

$w_2 = 0$, we obtain

$$V_f = s_f u' \tag{67}$$

Equation (67) results from the simplifying assumption which neglects column axial deformation and it is easy to see in this case from equations (64), (46) and (67) that the shear wall association (neglecting column axial deformation) would be equivalent to the plane association by means of pinned bars of a frame (not taking axial deformation into account) and one shear wall.

Let us now consider the shear wall association governing differential equation (11), rewritten as

$$\alpha(j_1 + j_2)u^{V} - j_c u''' = V - \alpha V'' \tag{68}$$

If one integration is performed, we obtain

$$\alpha(j_1 + j_2)u^{IV} - j_c u'' = \int_0^z V\mathrm{d}z - \alpha V' + C_1 = -M - \alpha V' + C_1 = -M - \alpha V' \tag{69}$$

Constant C_1 is zero, as one can conclude if every term of that equation is evaluated at $z = H$ and taking into consideration the boundary conditions (14·3), equation (18) and that $M(H) = 0$.

The formal likeness between equation (69) and the differential equation of the shear wall–frame association, repeated below

$$\frac{j_w}{s_f}u^{IV} - \left(1 + \frac{j_w}{j_f}\right)u'' = \frac{-V'}{s_f} - \frac{M}{j_f} \tag{70}$$

is perfect, if one makes in the preceding equation

$$j_w = j_g = j_1 + j_2; \tag{71·1}$$

$$s_f = s_g = \frac{3Ei}{2h}\frac{(2c)^2}{a^3}; \tag{71·2}$$

$$j_f = E\frac{(2c)^2}{\left(\frac{1}{A_1} + \frac{1}{A_2}\right)} \tag{71·3}$$

The formal coincidence between boundary conditions remains to be shown.

The first three conditions $u(0) = 0$; $u'(0) = 0$ and $u''(H)$ are the same in both problems.

The condition

$$V(H) = -j_w u'''(H) + s_f u'(H) - \frac{s_f}{E\ell_v^2}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_0^H (M - j_w u'')\mathrm{d}z \tag{72}$$

may be written as

$$V(H) = -j_w u'''(H) + \frac{s_f}{j_f}j_w[u'(H) - u'(0)] - \frac{s_f}{j_f}\int_0^H M\mathrm{d}z + s_f u'(H) \tag{73}$$

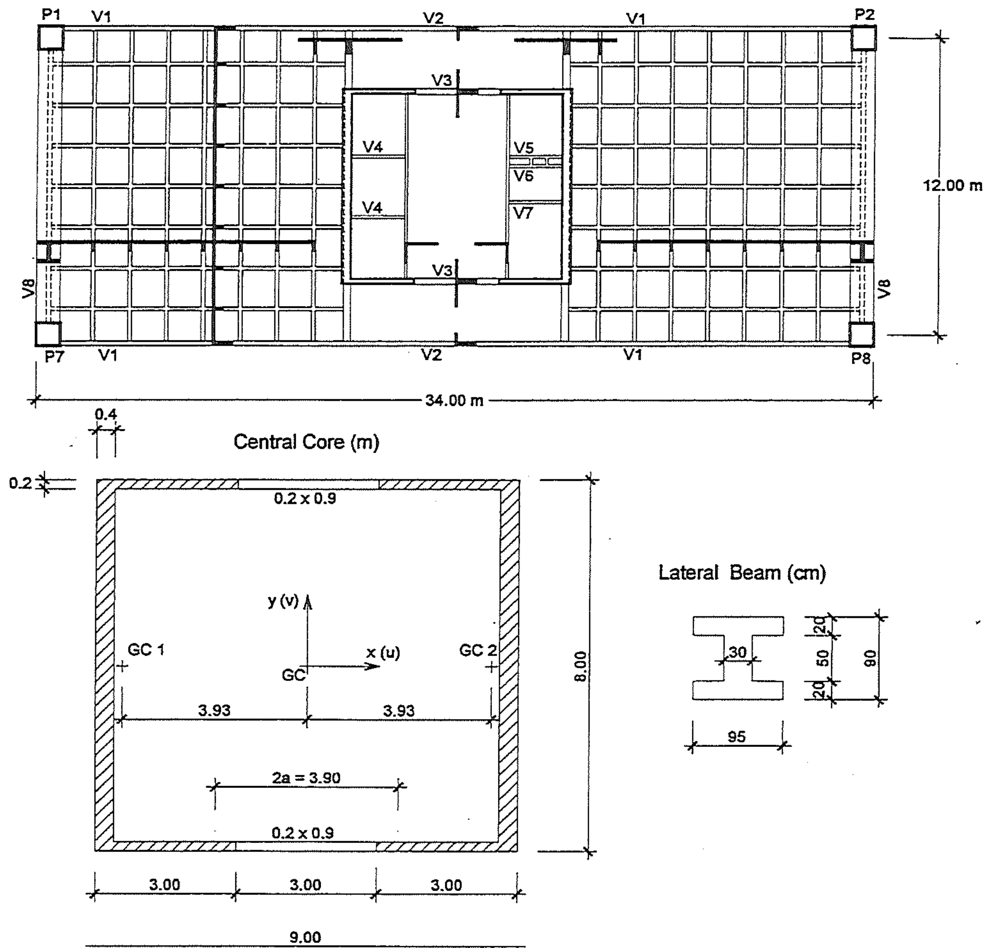

Figure 9. Plan view of the building; central core; lateral beam

or

$$-\frac{j_w}{s_f}u'''(H) + u'(H)\left[1 + \frac{j_w}{j_f}\right] = \frac{V(H)}{s_f} + \int_0^H \frac{M}{j_f}\mathrm{d}z \tag{74}$$

If one integrates equation (54) one will have

$$\frac{j_w}{s_f}u''' - \left(1 + \frac{j_w}{j_f}\right)u' = \frac{-V}{s_f} - \frac{\int_0^z M\,\mathrm{d}z}{j_f} + C_2 \tag{75}$$

Calculating (75) at $z = H$, considering condition (74), one concludes that $C_2 = 0$. Calculating now the expression (75) at the base ($z = 0$) will result in

$$u'''(0) = -\frac{V(0)}{j_w} \tag{76}$$

This last condition is coincident with boundary condition (16) for the shear wall association, here rewritten

$$u'''(0) = -\frac{V(0)}{E(I_1 + I_2)} = -\frac{V(0)}{j_g}$$

if one make the equivalence $j_w = j_g$

Then, one concludes that there does exist a structural behaviour equivalence, assured by the formal mathematical equivalence of the governing differential equations and their boundary conditions, for shear walls associated by lintel beams and plane frame–shear wall associations. Additionally, it is found that the necessary number of parameters to define the structural behaviour of these panels is three.

5. EXAMPLE

The application of the proposed approach can be illustrated through the analysis of the building shown in Figure 9, which is very similar to the 'Edifício Nova Paulista' designed by Mario Franco and J. Kassoy[3] (1974). Its basic data are: Building height (105 m); story height (3·5m); concrete Youngs modulus: 20000000 kN m^{-2}; wind loads: $q_x = 1{\cdot}2$kN m^{-2}, $q_y = 1{\cdot}5$kN m^{-2}. P1 = P2 = P7 = P8 = $0{\cdot}95 \times 0{\cdot}95$m^2.

In the x-direction the resistant structure is composed only of the two C-shaped core shear walls connected by lintel beams ($0{\cdot}20 \times 0{\cdot}90$ m^2) and in the y-direction the two pairs of parallel frames and shear walls can be considered to be interconnected by pinned horizontal bars. Then, as was proved in the preceding theoretical exposition, in the x and y directions, one has the two types of equivalent structures (shear walls associated by lintel beams and the plane frame–shear walls associated by pinned bars). For comparison purposes, computed top displacements are: $u = 0{\cdot}11753$ m and $v = 0{\cdot}31961$ m.

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THREE-DIMENSIONAL ASSOCIATION OF BENDING MOMENT AND SHEAR FORCE DEFORMABLE PANELS

EDDIE MANCINI* AND WALTER SAVASSI
Department of Structural Engineering, São Carlos School of Engineering-USP, Caixa Postal 359, 13566-590 São Carlos SP, Brazil

SUMMARY

In a previous paper, by using the continuous medium technique, the present authors showed that every plane panel, used to brace tall building structures acted upon by horizontal forces, is approximately equivalent to another panel composed of a shear wall joined to a two-column frame by pinned horizontal bars. In this paper, the three-dimensional association of such panels is considered. A simple example structure shows the advantage of the formulation, wherein very few parameters are needed to describe the whole structural behavior.

1. INTRODUCTION

During the last three decades, research on tall building structural behavior, using the continuous medium technique for analytical formulation has received frequent attention by the first-named author. As a consequence, some important conclusions about the adequacy of the use of that technique for obtaining wind action effects have been recorded, such as: (1) the advantageous simplicity of the mathematical model formulation, because such models become dependent on very few parameters; (2) the possibility of obtaining sufficiently accurate results, not only at the pre-design phase as observed many years ago, but also for effective use in the final structural design. Their use is possible today due to the refinement of the models, which can include many more details in order better to represent actual structures. In this way, in most cases, the resulting simple analytical models can be used without the need to employ computer programs. On the other hand, in order to be able to consider variable geometry and loading, or to define a path towards the utilization of a general computer program, it has also been shown by the work of the second-named author, that the combination of the equivalent continuous medium technique and the potentiality of the finite element method is an interesting tool for obtaining discrete model results that are also sufficiently accurate. Hence, as a result of the accumulated experience over so many years of using the continuous medium technique, one may conclude that it has great potential to become an applied design tool in the field of tall building structural engineering practice.

2. BENDING MOMENT AND SHEAR FORCE DEFORMABLE PANELS

Under this label one may consider basic panels such as frames with two or more columns, shear walls and even more general panels such as trussed-frames.

* Correspondence to: E. Mancini, Department of Structural Engineering, São Carlos School of Engineering-USP, Caixa Postal 359, 13566-590 São Carlos SP, Brazil.

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In this paper, the basic frame panel is used explicitly in the formulation and the resulting model is very close to the actual structure. The resulting theory may also be applied to other types of panel, if the appropriate parameters are properly derived. The stiffness parameter for the case of a two-column frame is derived in the Appendix, and the formula is also extended to the case of a frame with more than two columns.

It was shown in a previous paper (Mancini and Savassi, 1999) that the horizontal displacement components of a plane frame due to lateral force actions in the direction parallel to the beam axis $\overline{x}$, might be approximately calculated by

$$\overline{u}_f = \overline{u}_{fv} + \overline{u}_{fm} \tag{1}$$

where $\overline{u}_{fv}$ and $\overline{u}_{fm}$ are the parts of such displacement, respectively, due to shear force and bending moment deformations (resulting from axial column deformations). Indices f, v and m are related to frame, shear force and bending moment, respectively.

It is possible to show that, if V_f is the shear force and M_f is the bending moment due to lateral forces, then

$$\overline{u}'_{fv} = \frac{V_f}{s_f} \tag{2}$$

and

$$\overline{u}''_{fm} = \frac{M_f}{j_f} \tag{3}$$

where s_f is the frame shear force stiffness and j_f is a bending moment stiffness EI_f, with I_f indicating the second moment of the area of the horizontal global cross-section of the columns with respect to their common geometric center (neglecting the second moment of the area of the individual columns in relation to their own geometric centers). E is the material longitudinal modulus of elasticity.

For the case of a two-column frame panel one obtains

$$I_f = \frac{\ell_v^2}{\left(\frac{1}{A_1} + \frac{1}{A_2}\right)} \tag{4}$$

where A_1 and A_2 are the cross-sectional areas of each column, ℓ_v is the distance between their axes and, as shown in the Appendix,

$$s_f = \frac{12E}{h} \sum_{n.s} \left[k_{c.n} \frac{\sum_{b.n} k}{\sum_{m.n} k} \right] \tag{5}$$

Taking account of equations (1), (2) and (3), lateral displacements can be calculated by

$$\overline{u}_f = \int_o^z \frac{V_f}{s_f} \, dz + \int_o^z \int_o^z \frac{M_f}{j_f} \, dz \, dz \tag{6}$$

Considering the infinitesimal panel element, as shown in Figure 1, by imposing horizontal force

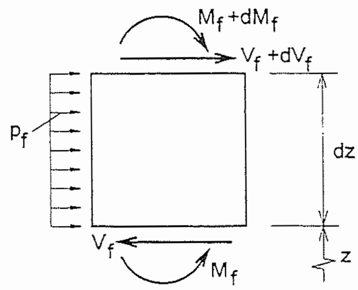

Figure 1. Panel (frame) infinitesimal element

equilibrium we obtain

$$p_f = -\frac{\mathrm{d}V_f}{\mathrm{d}z} \tag{7}$$

and by rotational equilibrium, one may write

$$V_f = -\frac{\mathrm{d}M_f}{\mathrm{d}z} \tag{8}$$

3. THREE-DIMENSIONAL ASSOCIATION OF PANELS

Differentiating equation (6), one has for the generic panel i, dropping index f in what follows,

$$\overline{u}'_i = \frac{V_i}{s_i} + \int_0^z \frac{M_i}{j_i}\,\mathrm{d}z \tag{9}$$

A second differentiation gives

$$\overline{u}''_i = -\frac{p_i}{s_i} + \frac{M_i}{j_i} \tag{10}$$

where p_i, V_i and M_i denote for panel i the distributed horizontal force, shear force and bending moment at level z, respectively.

One should observe that equation (6) is also valid for shear wall panels, i.e. panels mainly deformable by bending moment, when, additionally, one may also take shear force deformations into account.

Consider now the global axis reference system Oxyz, with vertical axis Oz, origin located at the building base, as shown in Figure 2. In that figure are also indicated the general projection of panel i with its local $\overline{x}_i$ axis and the projection of the vertical plane π of the horizontal external forces.

At the intersection of the vertical force plane π and the horizontal plane, a unit vector is considered whose horizontal and vertical components are, respectively, a and b, and c is the moment of the unit vector around the z-axis (positive in the counter-clockwise sense). In the same way, at the intersection of the generic panel i and the horizontal base plane another unit vector is considered, with components a_i and b_i, and moment c_i around the z-axis.

By hypothesis, rigid diaphragms, continuously distributed along the building height, interconnect-

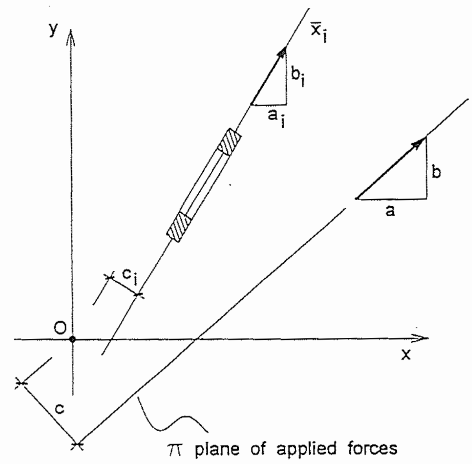

Figure 2. Coordinate systems for panel i and the applied forces plane

ing the actual vertical panels, replace floor slabs, which are assumed to be rigid in their own horizontal planes.

Equilibrium equations for a generic diaphragm at level z can be written

$$\begin{aligned} \sum p_i a_i &= pa \\ \sum p_i b_i &= pb \\ \sum p_i c_i &= pc \end{aligned} \tag{11}$$

where p is the lateral distributed force acting on the whole structure through the force plane π.

If V and M denote, respectively, the shear force and the bending moment absolute values due to external loading at level z, then the following equilibrium equations are also valid for the part of the building above level z:

$$\begin{aligned} \sum V_i a_i &= Va \\ \sum V_i b_i &= Vb \\ \sum V_i c_i &= Vc \end{aligned} \tag{12}$$

and

$$\begin{aligned} \sum M_i a_i &= Ma \\ \sum M_i b_i &= Mb \\ \sum M_i c_i &= Mc \end{aligned} \tag{13}$$

Equation (10) may be written in the form

$$s_i \bar{u}''_i = -p_i + \frac{s_i}{j_i} M_i \tag{14}$$

or, if

$$\lambda_i = \frac{s_i}{j_i} \tag{15}$$

we have

$$s_i \overline{u}''_i = -p_i + \lambda_i M_i \tag{16}$$

Multiplying every term of that equation by a_i and summing gives

$$\sum s_i \overline{u}''_i a_i = -\sum p_i a_i + \sum \lambda_i M_i a_i \tag{17}$$

The displacements of a generic diaphragm are called u and v, respectively, along the Ox and Oy global axes, together with its rotation w around the Oz-axis.

Cinematic compatibility, imposed by the rigid diaphragms at horizontal planes, gives

$$\overline{u}_i = a_i u + b_i v + c_i w \tag{18}$$

Substituting this equation in equation (17) gives

$$\sum s_i (a_i u'' + b_i v'' + c_i w'') a_i = -\sum p_i a_i + \sum \lambda_i M_i a_i \tag{19}$$

By the first of equations (11), and defining

$$S_{gh} = \sum s_i g_i h_i \tag{20}$$

where g and h represent any one of the coordinates a_i, b_i or c_i, one obtains

$$S_{aa} u'' + S_{ab} v'' + S_{ac} w'' = -pa + \sum \lambda_i M_i a_i \tag{21}$$

A similar procedure with the remaining coordinates b_i and c_i will give the following set of equations:

$$[S] \begin{pmatrix} u'' \\ v'' \\ w'' \end{pmatrix} = -p \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} \sum \lambda_i M_i a_i \\ \sum \lambda_i M_i b_i \\ \sum \lambda_i M_i c_i \end{pmatrix} \tag{22}$$

where

$$[S] = \begin{bmatrix} S_{aa} & S_{ab} & S_{ac} \\ S_{ab} & S_{bb} & S_{bc} \\ S_{ac} & S_{bc} & S_{cc} \end{bmatrix} \tag{23}$$

Note that equation (10) can also be written in the following form, which is more appropriate when dealing with higher λ_i values, as in the case of shear walls:

$$j_i\overline{u}_i'' = -\lambda_i^* p_i + M_i \tag{24}$$

where

$$\lambda_i^* = \frac{1}{\lambda_i} = \frac{j_i}{s_i} \tag{25}$$

From there, proceeding in the same way as before, the following alternative form would be obtained:

$$[J]\begin{pmatrix} u'' \\ v'' \\ w'' \end{pmatrix} = M\begin{pmatrix} a \\ b \\ c \end{pmatrix} - \begin{pmatrix} \sum \lambda_i^* p_i a_i \\ \sum \lambda_i^* p_i b_i \\ \sum \lambda_i^* p_i c_i \end{pmatrix}. \tag{26}$$

where, the matrix $[J]$, according with the notation

$$J_{gh} = \sum j_i g_i h_i \tag{27}$$

where g and h represent any of the coordinates a_i, b_i or c_i, is given by

$$[J] = \begin{bmatrix} J_{aa} & J_{ab} & J_{ac} \\ J_{ab} & J_{bb} & J_{bc} \\ J_{ac} & J_{bc} & J_{cc} \end{bmatrix} \tag{28}$$

From equation (10), and combining equations (7) and (8), we have

$$\overline{u}_i'' = -\frac{1}{s_i}\left(\frac{\mathrm{d}^2 M_i}{\mathrm{d}z^2}\right) + \frac{M_i}{j_i} \tag{29}$$

If the preceding expression is written for $i \geq 4$, and using the second derivative of equation (18) together with equation (22), one obtains

$$\begin{pmatrix} \overline{u}_4'' \\ \overline{u}_5'' \\ --- \\ \overline{u}_n'' \end{pmatrix} = \begin{pmatrix} -\frac{1}{s_4}\left(\frac{\mathrm{d}^2 M_4}{\mathrm{d}z^2}\right) + \frac{M_4}{j_4} \\ -\frac{1}{s_5}\left(\frac{\mathrm{d}^2 M_5}{\mathrm{d}z^2}\right) + \frac{M_5}{j_5} \\ ---------- \\ -\frac{1}{s_n}\left(\frac{\mathrm{d}^2 M_n}{\mathrm{d}z^2}\right) + \frac{M_n}{j_n} \end{pmatrix} = \begin{pmatrix} a_4\ b_4\ c_4 \\ a_5\ b_5\ c_5 \\ ---- \\ a_n\ b_n\ c_n \end{pmatrix}\begin{pmatrix} u'' \\ v'' \\ w'' \end{pmatrix}$$
$$= \begin{pmatrix} a_4\ b_4\ c_4 \\ a_5\ b_5\ c_5 \\ ---- \\ a_n\ b_n\ c_n \end{pmatrix}[S]^{-1}\left[-p\begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} \sum \lambda_i M_i a_i \\ \sum \lambda_i M_i b_i \\ \sum \lambda_i M_i c_i \end{pmatrix}\right] \tag{30}$$

Obviously

$$\begin{pmatrix} \sum \lambda_i M_i a_i \\ \sum \lambda_i M_i b_i \\ \sum \lambda_i M_i c_i \end{pmatrix} = \begin{pmatrix} \lambda_1 a_1 \cdot \lambda_2 a_2 \cdot \lambda_3 a_3 \cdot \lambda_4 a_4 \cdots \lambda_n a_n \\ \lambda_1 b_1 \cdot \lambda_2 b_2 \cdot \lambda_3 b_3 \cdot \lambda_4 b_4 \cdots \lambda_n b_n \\ \lambda_1 c_1 \cdot \lambda_2 c_2 \cdot \lambda_3 c_3 \cdot \lambda_4 c_4 \cdots \lambda_n c_n \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ --- \\ M_4 \\ \vdots \\ M_n \end{pmatrix} \tag{31}$$

The equilibrium equations (13) may be expressed as

$$\begin{pmatrix} a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5 \cdots a_n \\ b_1 \cdot b_2 \cdot b_3 \cdot b_4 \cdot b_5 \cdots b_n \\ c_1 \cdot c_2 \cdot c_3 \cdot c_4 \cdot c_5 \cdots c_n \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ --- \\ M_4 \\ M_5 \\ \vdots \\ M_n \end{pmatrix} = M \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{32}$$

or

$$[A_{1-3}] \begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} + [A_{4-n}] \begin{pmatrix} M_4 \\ M_5 \\ \vdots \\ M_n \end{pmatrix} = M \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{33}$$

Then, pre-multiplying the preceding equation by the inverse of the $[A_{1-3}]$ matrix will give

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = -[A_{1-3}]^{-1}[A_{4-n}] \begin{pmatrix} M_4 \\ M_5 \\ \vdots \\ M_n \end{pmatrix} + M[A_{1-3}]^{-1} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{34}$$

Expression (31) can be written as

$$\begin{pmatrix} \sum \lambda_i M_i a_i \\ \sum \lambda_i M_i b_i \\ \sum \lambda_i M_i c_i \end{pmatrix} = [LA_{1-3}] \begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} + [LA_{4-n}] \begin{pmatrix} M_4 \\ M_5 \\ \vdots \\ M_n \end{pmatrix} \tag{35}$$

Substituting equation (34) into equation (35) gives

$$\begin{pmatrix} \sum \lambda_i M_i a_i \\ \sum \lambda_i M_i b_i \\ \sum \lambda_i M_i c_i \end{pmatrix} = [LA_{1-3}] \left\{ -[A_{1-3}]^{-1}[A_{4-n}] \begin{pmatrix} M_4 \\ M_5 \\ \vdots \\ M_n \end{pmatrix} + M[A_{1-3}]^{-1} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\} + [LA_{4-n}] \begin{pmatrix} M_4 \\ M_5 \\ \vdots \\ M_n \end{pmatrix} \tag{36}$$

Substitution of equation (36) into equation (30) will produce a system of second-order differential equations in (M_4, M_5, ..., M_n).

The boundary conditions for that system are as follows.

(a) Zero bending moments at the top of every panel, i.e. $M_i\ (H) = 0$; in particular

$$\begin{aligned} M_4(H) &= 0 \\ M_5(H) &= 0 \\ &- - - - - \\ M_n(H) &= 0 \end{aligned} \tag{37}$$

where H is the total building height.

(b) Evaluating expression (9) at the base ($z = 0$) one has

$$\overline{u}_i'(0) = \frac{V_i(0)}{s_i} \tag{38}$$

or

$$s_i(a_i u'(0) + b_i v'(0) + c_i w'(0)) = V_i(0) \tag{39}$$

Multiplying both sides of this equation by a_i and summing will give, taking account of the equilibrium equations (12),

$$S_{aa}u'(0) + S_{ab}v'(0) + S_{ac}w'(0) = V(0)a \tag{40}$$

Proceeding in the same way with the b and c coordinates will give

$$\begin{pmatrix} S_{aa} & S_{ab} & S_{ac} \\ S_{ab} & S_{bb} & S_{bc} \\ S_{ac} & S_{bc} & S_{cc} \end{pmatrix} \begin{pmatrix} u'(0) \\ v'(0) \\ w'(0) \end{pmatrix} = V(0) \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{41}$$

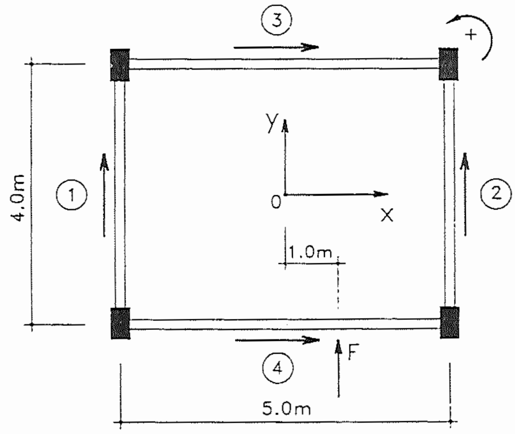

Figure 3. Building plan-view

from which one obtains

$$\begin{pmatrix} u'(0) \\ v'(0) \\ w'(0) \end{pmatrix} = [S]^{-1} V(0) \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{42}$$

Then, for the generic panel i, we have

$$V_i(0) = -M'_i(0) = s_i \bar{u}'_i(0) = s_i[a_i u'(0) + b_i v'(0) + c_i w'(0)] \tag{43}$$

Conditions (43) for $i \geq 4$ are the remaining boundary conditions for the system of differential equations that solve the problem.

4. EXAMPLE

In order to exemplify the application of the exposed theory, the structure shown in Figure 3 is considered. It is formed by 4 frames, with 20 floors, 3·0 m apart, total height of 60 m. All the columns have rectangular (30 cm × 50 cm) cross-section and all the horizontal beams have rectangular (20 cm × 50 cm) cross-section. The material is reinforced concrete with longitudinal modulus of elasticity $E = 200{\cdot}000$ kN dm^{-2}.

Plan view dimensions are also shown in Figure 3, where frames are indicated by encircled numbers.

Arrows indicate assumed positive directions associated with respective frame displacements and forces. The Oxyz coordinate system has its origin O point at the geometric center of the building base. Loading is due to a concentrated force $F = 10$kN applied at the top slab level in the position indicated in Figure 3, where the positive direction for moments and rotations is also shown.

In order to calculate s_f parameter for each frame, the following expression was used:

$$s_f = \frac{24Ek_p k_v}{h(2k_p + k_v)}$$

which is valid for frames with twin columns.

Table I. Frame characteristic parameters

Frame	a_i	b_i	c_i (dm)	s_i (kN)	j_i (kN dm^{-2})
1	0·0	+1·0	−25	33,333.	$2{\cdot}4 \times 10^9$
2	0·0	+1·0	+25	33,333.	$2{\cdot}4 \times 10^9$
3	+1·0	0·0	−20	21,429.	$3{\cdot}75 \times 10^9$
4	+1·0	0·0	+20	21,429.	$3{\cdot}75 \times 10^9$

To calculate $j_f = EI_f$, expression (4) was used.Table I contain all the parameter numerical values (in kN and dm).

Loading coordinates are, for this case.

$$a = 0{\cdot}0; \; b = 1{\cdot}0; \; c = +10$$

The system of differential equations, in this case, reduces to a single differential equation in M_4, as follows:

$$-4{\cdot}6666 \times 10^{-5} M''_4 + 3{\cdot}7787 \times 10^{-10} M_4 = -3{\cdot}4008 \times 10^{-6} p + 4{\cdot}7234 \times 10^{-11} M$$

In this example, one has

$$p = 0; \; M = F(H - z) = 10(600 - z)$$

and the differential equation becomes

$$-4{\cdot}6666 \times 10^{-5} M''_4 + 3{\cdot}7787 \times 10^{-10} M_4 = 4{\cdot}7234 \times 10^{-11}(6000 - 10z)$$

The first boundary condition is

$$M_4(H) = M_4(600) = 0$$

The second boundary condition is obtained from

$$\bar{u}'_4(0) = \frac{V_4(0)}{s_{f4}} = (a_4 \quad b_4 \quad c_4)[S]^{-1} V(0) \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

and hence

$$V_4(0) = -M'_4(0) = 0{\cdot}72876 \text{ kN}$$

For panel 4, the resulting bending moment is

$$M_4 = 5{\cdot}829 e^{2{\cdot}8456 \times 10^{-3} z} - 177{\cdot}26 e^{-2{\cdot}8456 \times 10^{-3} z} + 749{\cdot}99 - 1{\cdot}25z$$

and for shear force

$$V_4 = -M'_4$$

By using the global equilibrium equations one may obtain

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \begin{pmatrix} 0\cdot 8 \\ -0\cdot 8 \\ -1\cdot 0 \end{pmatrix} M_4 + \begin{pmatrix} 0\cdot 3 \\ 0\cdot 7 \\ 0 \end{pmatrix} M$$

and therefore

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0\cdot 8 \\ -0\cdot 8 \\ -1\cdot 0 \end{pmatrix} V_4 + \begin{pmatrix} 0\cdot 3 \\ 0\cdot 7 \\ 0 \end{pmatrix} V$$

For the building displacements evaluation, referred to the global coordinate system, expression (22) is used, leading to

$$u'' = 0$$

$$v'' = 2\cdot 08335 \times 10^{-10} M$$

$$w'' = -5\cdot 560 \times 10^{-12} M_4 + 2\cdot 362 \times 10^{-12} M$$

whose boundary conditions are

$$u(0) = 0;\; v(0) = 0;\; w(0) = 0$$

and

$$\begin{pmatrix} u'(0) \\ v'(0) \\ w'(0) \end{pmatrix} = [S]^{-1} V(0) \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0\cdot 15 \times 10^{-3} \\ 0\cdot 17 \times 10^{-5} \end{pmatrix}$$

For the resulting displacements we have

$$u \equiv 0$$

$$v = 2\cdot 08335 \times 10^{-10}(3000z^2 - 1\cdot 6666z^3) + 0\cdot 15 \times 10^{-3} z$$

$$\begin{aligned} w = {} & -5\cdot 560 \times 10^{-12}\left(\frac{5\cdot 829}{(2\cdot 8456 \times 10^{-3})^2}\, e^{2\cdot 8456\times 10^{-3} z} \right. \\ & - \frac{177\cdot 26}{(2\cdot 8456 \times 10^{-3})^2} e^{-2\cdot 8456\times 10^{-3} z} + \frac{749\cdot 99}{2} z^2 \\ & \left. - \frac{0\cdot 625}{3} z^3 \right) + 2\cdot 362 \times 10^{-12}\left(3000z^2 - \frac{5}{3} z^3\right) \\ & + 0\cdot 20581 \times 10^{-5} z - 1\cdot 1771 \times 10^{-4} \end{aligned}$$

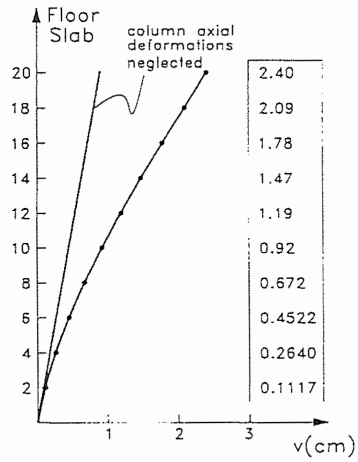

Figure 4. Horizontal displacements v

The results are shown graphically in Figures 4 to 11. Figure 4 displays the horizontal displacements v of the slabs in the Oyz-plane, both taking account of frame column axial deformations and neglecting such deformations. Slabs rotations are represented in Figure 5. Figures 6 to 8 show the shear forces on panels 3, 4, 2 and 1. The corresponding graphics are limited to the region defined by vertical lines representing panel shear forces that would appear if they were assumed to be deformable solely by shear forces (rigid to bending) and deformable solely by bending moments (rigid to distortion).

The bending moments experienced by the panels are indicated in Figures 9 to 11. Their values are also graphically limited by the particular extreme cases considered in the preceding paragraph.

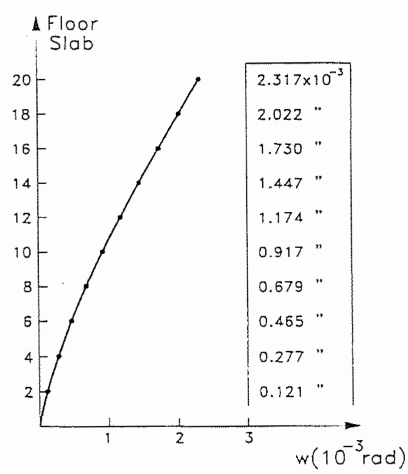

Figure 5. Floor slab rotations

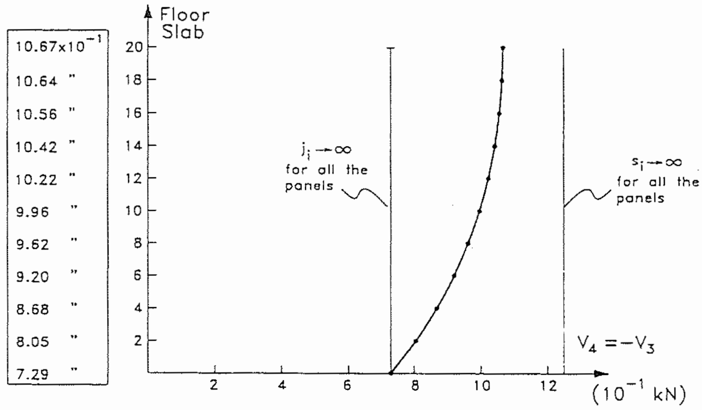

Figure 6. Panel 4 shear forces

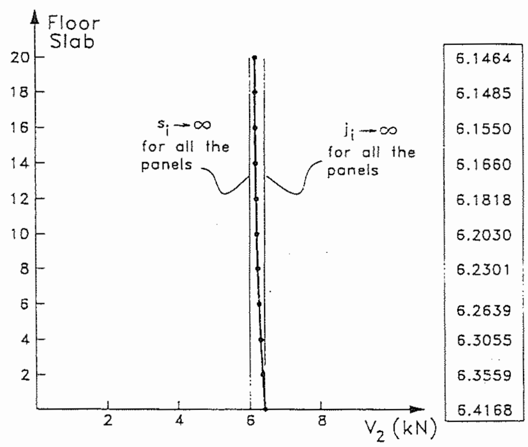

Figure 7. Panel 2 shear forces

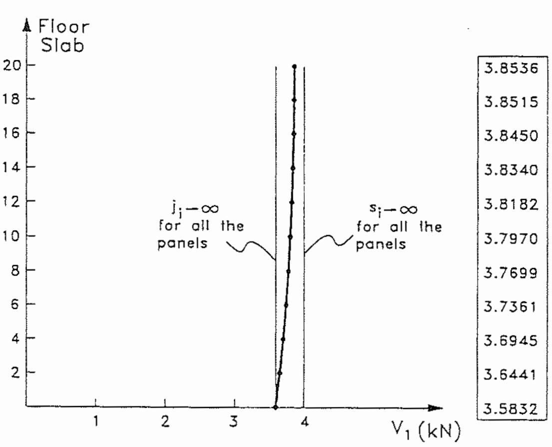

Figure 8. Panel 1 shear forces

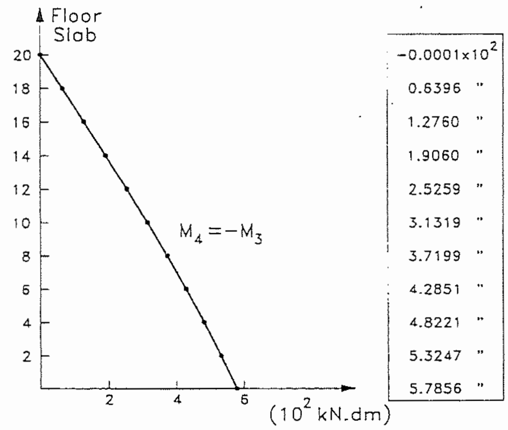

Figure 9. Panel 4 bending moments

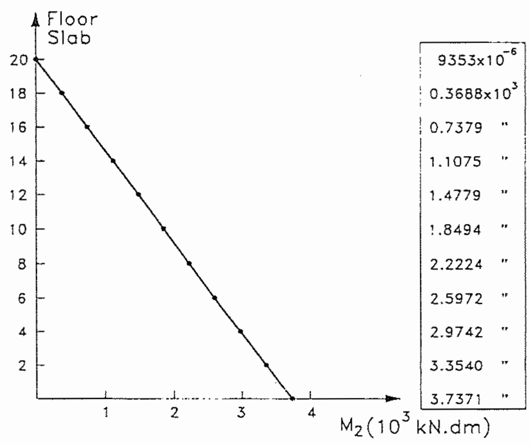

Figure 10. Panel 2 bending moments

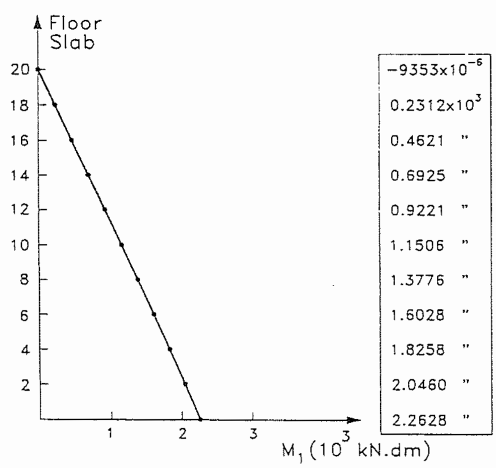

Figure 11. Panel 1 bending moments

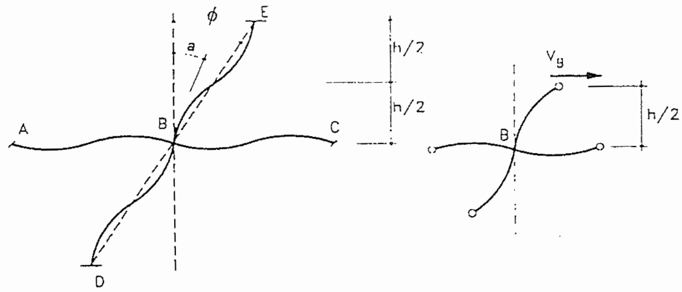

Figure 12. Frame stiffness

APPENDIX

Frame shear force stiffness: coefficient s_f

Regular frames acted upon by lateral loading are supposed to have zero bending moment points located at beam and column midpoints. This is equivalent to assuming equal rotations for all the beam nodes and that the rotations of consecutive nodes of the columns are very close together.

Figure 12 shows the frame node B and its neighboring nodes A, C, E, D. The story distortion and the node B rotations are called ϕ and a, respectively. The bending moments applied by the bars to node B, which are positive when in the clockwise direction, are given by

$$\begin{aligned} M_{BE} &= 6Ek_E(\phi - a) \\ M_{BD} &= 6Ek_D(\phi - a) \\ M_{BA} &= -6Ek_A a \\ M_{BC} &= -6Ek_C a \end{aligned} \tag{A1}$$

where letter k represent the bar stiffness $\frac{I_i}{\ell_i}$.

Nodal equilibrium gives

$$a = \frac{k_E + k_D}{k_A + k_C + k_E + k_D} \cdot \phi \tag{A2}$$

The shear force of column B is

$$V_B = \frac{2M_{BE}}{h} \tag{A3}$$

Then, by equations (A1) and (A2) it follows that

$$V_B = \frac{12E}{h} k_E \frac{\sum_b k}{\sum_m k} \phi \tag{A4}$$

where indices b and m indicate summation extended to beams and to the totality of members that are concurrent at that node, respectively.

Then, the column B stiffness is

$$S_B = \frac{12E}{h} k_E \frac{\sum_b k}{\sum_m k} \tag{A5}$$

To obtain the frame stiffness, one has to add each column's contribution, resulting in

$$s_f = \frac{12E}{h} \sum_{n,s} \left[k_{c,n} \frac{\sum_{b,n} k}{\sum_{m,n} k} \right] \tag{A6}$$

where

k is the beam or column relation (I/ℓ),
I denotes the second moment of area,
$\sum_{n,s}$ is the sum extended to all the nodes of the story under consideration,
$k_{c,n}$ is the relation I/ℓ for the column above the considered node,
$\sum_{m,n} k$ is the sum extended to all the members that are concurrent at the node,
$\sum_{b,n} k$ is the sum extended to all the beams that are concurrent at the node,
E is the material longitudinal modulus of elasticity, h is the story height.

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ONE-DIMENSIONAL FINITE ELEMENT SOLUTION FOR TALL BUILDING STRUCTURES UNIFIED PLANE PANELS FORMULATION

WALTER SAVASSI[1]* AND EDDIE MANCINI
Department of Structural Engineering, São Carlos School of Engineering–USP, São Carlos, Brazil

SUMMARY

In a previous paper (Mancini and Savassi, 1999), it was shown that every plane panel, used to brace tall building structures, can be easily and generally approached through the use of the continuous medium technique (CMT) (Albigés and Goulet, 1960). In that paper, following a so-called local formulation, i.e., by deriving the governing differential equations system of the panel, in terms of u(z) panel horizontal displacement and w_i columns or walls axial displacements, the equivalence (likeness) of formal mathematics, and hence of structural behaviour, between the panel composed by *a pair of shear walls associated by lintel beams* and another panel formed by the plane association, by pinned horizontal bars, of *one shear wall and one single bay frame*, was also shown. In both cases, axial deformations due to axial forces on vertical members were taken into account. In this paper, confirming those conclusions, but now following a global formulation (i.e., considering the total potential energy of each panel: strain energy plus applied load potentials), the mathematical equivalence between those two types of plane panels is again revealed by comparison of their two total potential energy analytical expressions. Additionally, based on that variational approach, the one-dimensional finite element formulation is presented. This enlarges the possibilities of solutions for more general types of panels, like those with variable geometry or loading, without any further difficulty. The procedure, for any type of panel, can be codified in one single computer program, very similar to those used to solve plain continuous beam problems. Copyright © 2004 John Wiley & Sons, Ltd.

1. INTRODUCTION

CMT is an approximate structural analysis technique. When using the CMT, very few parameters are needed to define any structural panel, both in relation to its stiffness or behaviour characteristics when acted upon by lateral wind forces. So, it is of great value to take advantage of that, both analytically or numerically.

As is well known, the corresponding total potential energy analytical expression of each panel type is called a functional. In structural mechanics, for the cases here considered, the functions u(z) and $w_i(z)$ which minimize the corresponding functional are the solution for each panel. These functions are not directly obtainable in the analytical minimization procedure. Instead, they are solutions for the minimizing conditions, which are the so-called Euler equations for the functional, i.e. the differential equation system and the corresponding boundary conditions.

In certain cases, it is possible to construct by inspection the functional whose minimizing conditions are a known differential equation (or system) and its boundary conditions. Consider two plane

*Correspondence to: Walter Savassi, Department of Structural Engineering, São Carlos School of Engineering–USP, Caixa Postal 359, 135666-590 São Carlos SP, Brazil. E-mail savassi@sc.usp.br

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panels, one composed by *two shear walls associated by lintel beams* and another formed by *a shear wall connected to a frame by pinned bars.* Mancini and Savassi (1999) proved the mathematical equivalence, and hence of structural behaviour, between these panels by comparing their governing differential equations and boundary conditions (local formulation). Now, in this paper, the same mathematical likeness is again revealed by constructing the two functionals (global formulation) which correspond to the two above-mentioned plane panels. As will be seen in this approach, it turned out that these functionals are *formally similar* and hence they have similar solution, given by the same type of the Euler equations.

Then, if a finite element approximation is introduced in any one of these functionals, numerical discrete approximate results can be found, even for more general cases of panels. One advantage of such a procedure is that always the same corresponding computer program can be used to solve any case of planar association, once the panel parameters are properly derived, by the use of the CMT. Another important advantage is that geometric and/or loading characteristics are free to vary along the building height. On the other hand, even with constant geometry and/or loading, when using the local procedure, based on the system of differential equations, the solution would require the calculation of constants whose expressions always depend on the boundary and loading conditions for each particular case.

This paper will begin with a preliminary detailed local formulation for three particular panels, leading to the governing differential equation for a *single shear wall* and to the system of differential equations for *two shear walls associated by lintel beams* and for the *plane frame* panels.

Then, one will construct three corresponding functionals, whose minimizing conditions are those above-mentioned differential equations (or systems). By adding (linear theory) the functional of one single shear wall, P_w, to the functional of a single frame, P_f, and comparing the resulting functional, P_{fw}, with one of the two shear walls associated with lintel beams, P_{ww}, the formal likeness between these functionals (P_{fw} and P_{ww}) will be evident. Whence, as a practical consequence, it will be possible to use the variational approach of finite element method to deal with this unified planar panel behaviour.

Considering the shear wall association as a basic case, for computer program codification purposes, one may introduce u(z), $w_1(z)$ and $w_2(z)$ shape function approximations and use all the well-known steps of finite element analysis to solve various types of plane panels.

2. SHEAR WALL

A shear wall is a plane panel deformable by bending moment action and extremely rigid to shear force action, hence having the following governing differential equation for the deflected curve u(z) of its vertical axis (see also Table 2 for notation):

$$j_w u^{iv} = p_w \tag{1}$$

where j_w (w-index standing for wall) is the bending rigidity modulus EI_w of the wall and p_w is the given external loading. Boundary conditions are:

$$u(0) = 0 \tag{2.1}$$

$$u'(0) = 0 \tag{2.2}$$

$$j_w u'''(H) = -V(H) \tag{2.3}$$

$$j_w u''(H) = M(H) \tag{2.4}$$

The assumed positive directions for internal actions are shown in Figure 1.

3. SERIAL SHEAR WALL ASSOCIATION

The system of differential equations is derived in terms of u(z) and $w_i(z)$, respectively horizontal and axial wall displacements for a serial shear wall association (Figure 2).

CMT assumes that an equivalent distributed force q_c (Figure 4) through the vertical z coordinate replaces the actual concentrated shear forces Q_c acting on the horizontal connecting beams. For convenience, in the foregoing preliminary *local approach* (looking for the differential equations), the structure is supposed to have uniform geometry and loading, and is rigidly built-in at the base. Afterwards, when a *global approach* will be used, with an approximated one-dimensional finite element panel discretization, together with its equivalent continuous medium, those characteristics will be free to vary.

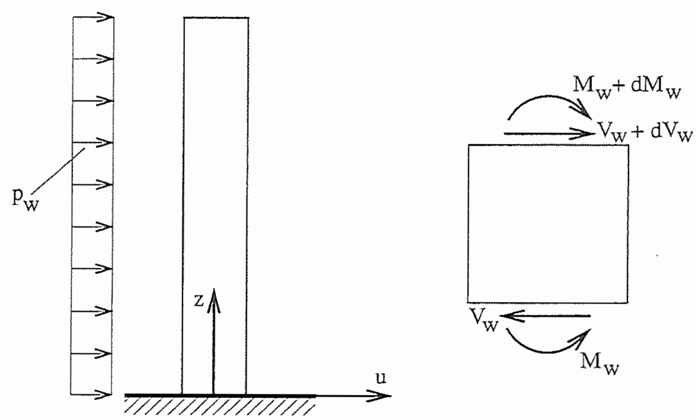

Figure 1. Shear wall

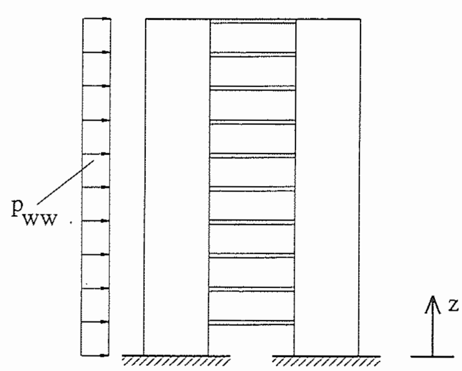

Figure 2. Shear walls connected by lintel beams

From displacements compatibility, considering panel deformation shown in Figure 3,

$$f^* + w_2 = w_1 + f \tag{3}$$

or, for the case of geometric linear behaviour (small rotations):

$$Lu' + w_2 = w_1 + \frac{Q_c \ell^3}{12EI_c} \tag{4}$$

In terms of the distributed diaphragm vertical shear force (Figure 4), i.e.

$$q_c = \frac{Q_c}{h} \tag{5}$$

Equation (4) may be written as

$$Lu' + w_2 - w_1 = \frac{q_c h \ell^3}{12EI_c} \tag{6}$$

from which we get

$$q_c = \frac{12EI_c}{h\ell^3}[Lu' - (w_1 - w_2)] = S[Lu' - (w_1 - w_2)] \tag{7}$$

Let us now consider the shear wall infinitesimal elements (dz) (Figure 4), and the corresponding internal forces. By hypothesis, as shown in Figure 3, the midpoints of lintel beam length are assumed to have no bending moment.

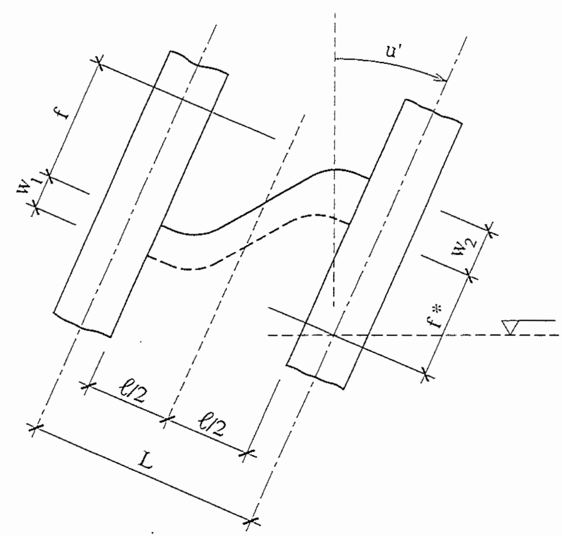

Figure 3. Panel deformations

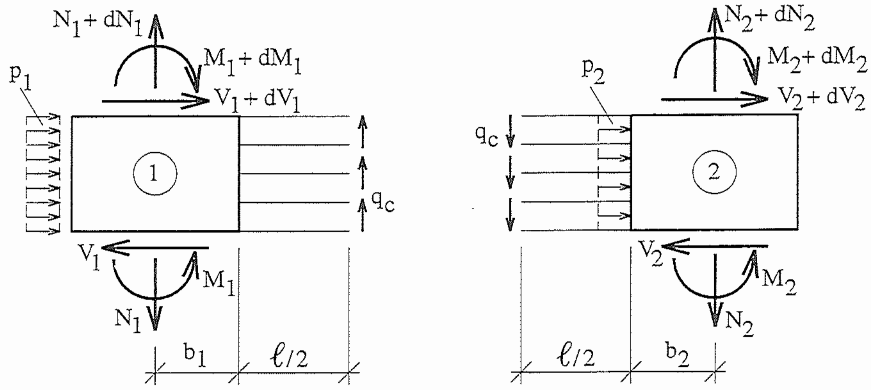

Figure 4. Shear wall infinitesimal elements

The following relation governs the deflected curve of each i[th] wall, neglecting axial and shear deformations of lintel beams:

$$u'' = \frac{M_i}{EI_{wi}} \tag{8}$$

and, by imposing rotational equilibrium to shear wall [1] infinitesimal element (Figure 4), one may write the equation:

$$V_1 = -EI_{w1}u''' + q_c\left(\frac{\ell}{2} + b_1\right) \tag{9}$$

Similarly for the shear wall [2] infinitesimal element:

$$V_2 = -EI_{w2}u''' + q_c\left(\frac{\ell}{2} + b_2\right) \tag{10}$$

and then, by summation, it follows:

$$V = -E(I_{w1} + I_{w2})u''' + Lq_c \tag{11}$$

Differentiating both sides of this equation will give

$$\frac{dV}{dz} = -E(I_{w1} + I_{w2})u^{iv} + Lq_c' \tag{12}$$

Differentiating Equation (7) and substituting it in Equation (12) (which also equals $p_{ww}(z) = p_1 + p_2$ for the case of an external distributed horizontal loading) will produce

$$j_g u^{iv} - SL^2u'' + SL(w_1' - w_2') = p_{ww} \tag{13}$$

By imposing vertical translational equilibrium to the shear wall [1] and [2] infinitesimal elements the following results:

$$\frac{dN_1}{dz} = -q_c \tag{14}$$

$$\frac{dN_2}{dz} = q_c \tag{15}$$

Relating axial forces to the corresponding axial deformations:

$$N_1 = EA_1 w_1' \tag{16}$$

$$N_2 = EA_2 w_2' \tag{17}$$

Then, using Equations (14), (15) and (7):

$$EA_1 w_1'' = -q_c = -S[Lu' - (w_1 - w_2)] \tag{18}$$

$$EA_2 w_2'' = q_c = S[Lu' - (w_1 - w_2)] \tag{19}$$

or

$$F_1 w_1'' + SLu' - S(w_1 - w_2) = 0 \tag{20}$$

$$F_2 w_2'' - SLu' + S(w_1 - w_2) = 0 \tag{21}$$

Equations (13), (20) and (21) solve the problem.
The boundary conditions (both essential and mechanical) are

(a) at the base (rigidly built in):

$$u(0) = 0 \tag{22.1}$$

$$u'(0) = 0 \tag{22.2}$$

$$w_1(0) = 0 \tag{22.3}$$

$$w_2(0) = 0 \tag{22.4}$$

(b) at the top:

$$j_g u''(H) = M(H)(= 0, \text{ if no external bending moment applied at the top}) \tag{22.5}$$

$$F_1 w_1'(H) = N_1(H)(= 0, \text{ if no external axial force applied at the top of wall 1}) \tag{22.6}$$

$$F_2 w_2'(H) = N_2(H)(= 0, \text{ if no external axial force applied at the top of wall 2}) \tag{22.7}$$

Using the global equilibrium Equation (11), evaluated at the top, after substitution of Equation (7), the following results (if no external concentrated horizontal force is applied at the top):

$$-j_g u'''(H) + SL^2 u'(H) - SL[w_1(H) - w_2(H)] = V(H) = 0 \tag{22.8}$$

4. PLANE FRAME

A twin columns plane frame analysis, under the action of lateral loading p_f, and considering the influence of their column axial deformations, is presented.

Figure 5 shows the deformed shape of one frame storey, one part of it due to the storey distortion and another one due to columns 1 and 2 axial deformations. β is the frame node rotation and u' is the rotation of the chord of the column deflected line. CMT assumes that adjacent storeys have the same u' and β deformations.

In the case where columns 1 and 2 have equal rigidities ($k_1 = k_2 = k_c$; $k_i = I_i/h$) and according to the Grinter signal rule (positive clockwise node rotations), the bending moments applied by the bars to node A are

$$M_{AC} = 6Ek_c(u' - \beta) \tag{23}$$

$$M_{AE} = 6Ek_c(u' - \beta) \tag{24}$$

$$M_{AB} = -6Ek_v\left[\beta - \frac{(w_1 - w_2)}{\ell_v}\right] \tag{25}$$

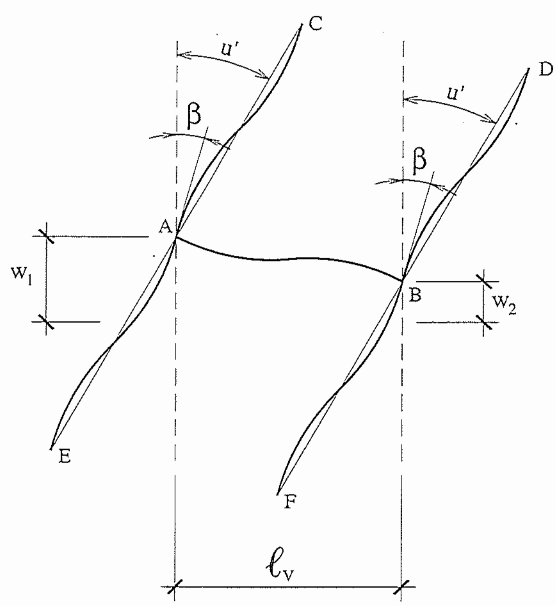

Figure 5. Deformation of a generic frame storey

The rotational equilibrium of node A will give

$$\beta = \frac{2k_c u' + k_v (w_1 - w_2)/\ell_v}{(2k_c + k_v)} \quad (26)$$

Assuming that the column midpoints (between adjacent stories) have zero bending moments, the frame horizontal shear force V_f can be written as

$$V_f = \frac{2}{h}(M_{AC} + M_{BD}) \quad (27)$$

Using Equation (23) and knowing that $M_{BD} = M_{AC}$, the following results:

$$V_f = \frac{24Ek_c}{h}(u' - \beta) \quad (28)$$

Substituting Equation (26) into Equation (28) will give

$$V_f = s_f u' - r_1 (w_1 - w_2) \quad (29)$$

where

$$s_f = \frac{24Ek_c k_v}{h(2k_c + k_v)} \quad (30)$$

$$r_1 = \frac{s_f}{\ell_v} \quad (31)$$

According with Figure 6, one has

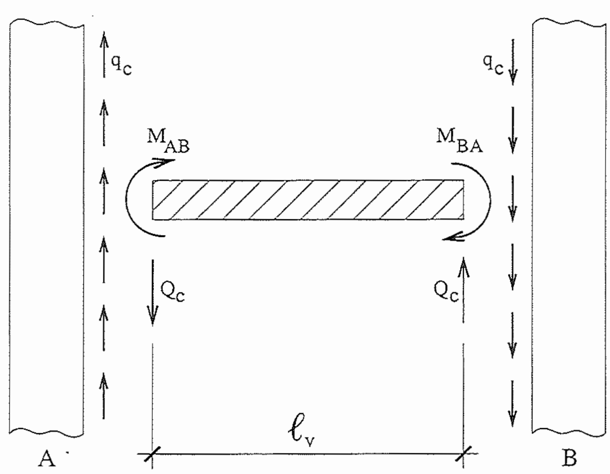

Figure 6. Internal actions on beam AB

$$q_c = \frac{1}{h\ell_v}(M_{AB} + M_{BA}) \tag{32}$$

and hence, from Equation (25),

$$q_c = r_1 u' - d_1(w_1 - w_2) \tag{33}$$

where

$$d_1 = \frac{r_1}{\ell_v} = \frac{s_f}{\ell_v^2} \tag{34}$$

Comparing Equations (29) and (33) the following results:

$$q_c = \frac{V_f}{\ell_v} \tag{35}$$

For a uniformly distributed horizontal external force $p_f(z)$:

$$V_f = p_f(H - z) \tag{36}$$

Differentiating Equations (29) and (36):

$$s_f u'' - r_1(w_1' - w_2') = -p_f \tag{37}$$

Figure 7 presents a generic infinitesimal element of column 1 and its respective loading q_c and internal actions.

Assuming as positive the directions shown in Figure 7, from the translation vertical equilibrium of column 1 generic element the following will result:

$$\frac{dN_1}{dz} = -q_c \tag{38}$$

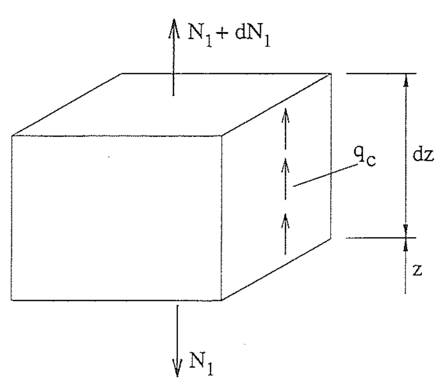

Figure 7. Column 1 generic infinitesimal element

As column 1 axial force is given by

$$N_1 = EA_{p1} w_1' \tag{39}$$

combining Equations (39) and (38) will give

$$EA_{p1} w_1'' = -q_c \tag{40}$$

Similarly, from the equilibrium of the column 2 infinitesimal element one gets

$$EA_{p2} w_2'' = q_c \tag{41}$$

Using Equation (33) in (40) and (41):

$$EA_{p1} w_1'' + r_1 u' - d_1 (w_1 - w_2) = 0 \tag{42}$$

$$EA_{p2} w_2'' - r_1 u' + d_1 (w_1 - w_2) = 0 \tag{43}$$

It is easy to see from Equations (40), (41) and (35) that, for calculations, the following form could be used:

$$EA_{p2} w_2'' = -EA_{p1} w_1'' = \frac{V_f}{\ell_v} \tag{44}$$

The differential equation system, composed of Equations (37), (42) and (43), solves the problem. The following boundary conditions should be used:

(a) at the base (z = o):

$$u(0) = 0 \tag{45.1}$$

$$w_1(0) = 0 \tag{45.2}$$

$$w_2(0) = 0 \tag{45.3}$$

(b) at the top (z = H), if no external vertical force is acting on the top of columns:

$$EA_{p1} w_1'(H) = 0 \quad \text{for } N_1(H) = 0 \tag{45.4}$$

$$EA_{p2} w_2'(H) = 0 \quad \text{for } N_2(H) = 0 \tag{45.5}$$

$$s_f u'(H) - r_1 [w_1(H) - w_2(H)] = V_f(H) (= 0, \text{ if } V_f(H) = 0) \tag{45.6}$$

The preceding results may be extended to frames with unequal columns, making

$$s_f = \frac{12E}{h} \sum_{ns} \left[k_{c.n} \frac{\sum_{b.n} k}{\sum_{m.n} k} \right] \tag{46}$$

where

k is the relation I/ℓ of the (members) beam or column under consideration;

$\sum_{n.s}$ is the summation extended to every *n*ode of the considered *s*torey;

$k_{c.n}$ is the relation I/ℓ of the *c*olumn above the considered *n*ode;

$\sum_{m.n} k$ is the summation extended to every concurrent *m*ember at the *n*ode;

$\sum_{b.n} k$ is the summation extended to every concurrent *b*eam at the *n*ode;

E is the material Young's modulus; and

h is storey height

Mancini and Savassi (1999), following a slightly different approach, at this point, have fully developed the few necessary details in order to obtain the proposed description of the equivalence between the shear wall association and the frame–shear wall connected by pinned bars. In this paper, as we are looking for that same mathematical equivalence, but following an energy approach, these details are omitted.

5. VARIATIONAL FORMULATION OF THE PROBLEM OF THE SHEAR WALL-FRAME ASSOCIATION BY PINNED BARS

Many of the building structures, acted by horizontal forces, can be analysed with the use of the model shown in Figure 8, represented by a frame linked to a shear wall by horizontal pinned bars, which are supposed to have no axial deformations. Those bars take the role of the floor slabs, supposed to be rigid to membrane (horizontal) actions. One example of such structures is given in Mancini and Savassi (1999).

As assumed by the continuous medium technique, the axially rigid horizontal connecting pinned bars are supposed to be replaced by a continuous medium, uniformly distributed through the panel

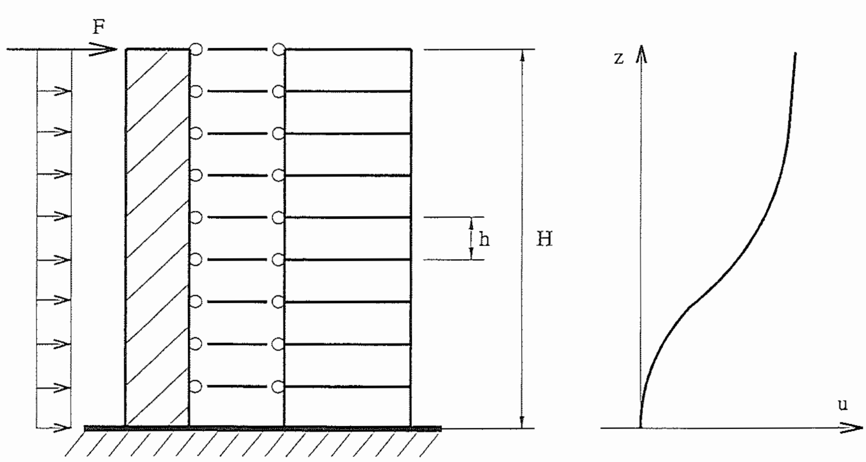

Figure 8. Frame–shear wall association by pinned bars

height. Under these and other previous already stated assumptions, the variational formulation is now considered.

It is possible to construct the respective functionals whose extremum conditions correspond to the differential equations systems for the basic structural panels here considered:

(a) the single shear wall: Equation (1);
(b) the plane frame: Equations (37), (42) and (43);
(c) the serial shear wall association: Equations (13), (20) and (21);

and their respective boundary conditions.

They are, for (a):

$$P_w = \frac{1}{2}\int_0^H [j_w u''^2 - 2up_w]dz \tag{47}$$

For (b):

$$P_f = \frac{1}{2}\int_0^H \left[s_f u'^2 - 2r_1 u'(w_1 - w_2) + d_1(w_1 - w_2)^2 + EAp_1 w_1'^2 + EAp_2 w_2'^2 - 2up_f\right]dz \tag{48}$$

For (c):

$$P_{ww} = \frac{1}{2}\int_0^H \left[j_g u''^2 + SL^2 u'^2 - 2SLu'(w_1 - w_2) + S(w_1 - w_2)^2 + F_1 w_1'^2 + F_2 w_2'^2 - 2up_{ww}\right]dz \tag{49}$$

Note that, at this point, different cross-sections for columns and walls were considered.

For continuously connected (frame and a shear wall) panels, $u_w = u_f = u$, ($p_w + p_f = p$, where p is the total external distributed force), and, for linear theory, it is easy to see that the functional P_{fw}, corresponding to the frame shear wall association by pinned bars, is given by the sum of Equations (48) and (47), i.e.:

$$P_{fw} = \frac{1}{2}\int_0^H \left[j_w u''^2 + s_f u'^2 - 2r_1 u'(w_1 - w_2) + d_1(w_1 - w_2)^2 + EA_{p1} w_1'^2 + EA_{p2} w_2'^2 - 2up\right]dz \tag{50}$$

Then, one can see that the functional P_{fw} for the frame shear wall association by pinned bars (Figure 8) has the same form as functional P_{ww} for the serial shear wall association by lintel beams, i.e.:

$$P_{fw} = P_f + P_w \approx P_{ww} \tag{51}$$

and they will coincide if one makes, for instance, in P_{ww}:

$$j_g = j_w \; ; SL^2 = s_f, (SL = r_1 \; ; S = d_1); F_1 = EA_{p1} \; ; F_2 = EA_{p2} \; ; p = p_{ww} \tag{52}$$

Then, comparing Equation (49) and Equation (50), one concludes that a structural behaviour equivalence does exist, assured by the formal mathematical equivalence of the corresponding functionals P_{ww} and P_{fw}. It is found too that only a few parameters are required to define the structural behaviour of these panels.

Supposing the serial shear wall association to be the basic reference case, if the variational formulation was to be used to find out under which conditions the corresponding functional P_{ww} would attain its extreme value (minimum in this particular case), the differential Equations system given by Equations (13), (20) and (21), together with the mechanical boundary conditions, would be obtained.

But, if one is much more interested in the finite element discrete numerical solution for such a problem, then this basic panel, together with its continuous medium, will be considered in the approximate minimizing procedure.

6. ONE-DIMENSIONAL FINITE ELEMENT SOLUTION

For the basic reference case of the serial shear wall association, the detailed discrete one-dimensional finite element formulation is now presented.

6.1 *Energy functional*

The total potential energy functional is given by Equation (49), now repeated:

$$P_{ww} = \frac{1}{2}\int_0^H \left[j_g u''^2 + SL^2 u'^2 - 2SLu'(w_1 - w_2) + S(w_1 - w_2)^2 + F_1 w_1'^2 + F_2 w_2'^2 - 2up_{ww} \right] dz$$
$$= \int_0^H \Im dz \qquad (49)$$

As is well known (Langhaar, 1962) part of the minimizing conditions for that functional, other than the appropriate boundary conditions, are the following equations:

(I)
$$\frac{\partial \Im}{\partial u} - \frac{d}{dz}\frac{\partial \Im}{\partial u'} + \frac{d^2}{dz^2}\frac{\partial \Im}{\partial u''} = 0 \qquad (53)$$

or

$$\frac{\partial \Im}{\partial u} = -p_{ww}; \quad \frac{\partial \Im}{\partial u'} = SL^2u' - SLw_1 + SLw_2; \quad \frac{\partial \Im}{\partial u''} = j_g u'' \qquad (54)$$

Then

$$j_g u^{iv} - SL^2 u'' + SL(w_1' - w_2') = p_{ww} \qquad (55)$$

(II)
$$\frac{\partial \Im}{\partial w_1} - \frac{d}{dz}\frac{\partial \Im}{\partial w_1'} = 0 \qquad (56)$$

or

$$\frac{\partial \Im}{\partial w_1} = -SLu' - Sw_2 + Sw_1; \quad \frac{\partial \Im}{\partial w_1'} = F_1 w_1' \qquad (57)$$

Then

$$F_1 w_1'' + SLu' - S(w_1 - w_2) = 0 \tag{58}$$

(III)

$$\frac{\partial \Im}{\partial w_2} - \frac{d}{dz}\frac{\partial \Im}{\partial w_2'} = 0 \tag{59}$$

or

$$\frac{\partial \Im}{\partial w_2} = SLu' - Sw_1 + Sw_2 \; ; \; \frac{\partial \Im}{\partial w_2'} = F_2 w_2' \tag{60}$$

Then

$$F_2 w_2'' - SLu' + S(w_1 - w_2) = 0 \tag{61}$$

Equations (55), (58) and (61) are the same as Equations (13), (20) and (21).

Essential boundary conditions are given by Equations (22.1) to (22.4) and, as can be shown, natural boundary conditions also arise from the minimizing procedure, and are given by Equations (22.5) to (22.8).

6.2 *Numerical model: cubic one-dimensional finite elements*

Suppose cubic interpolation functions for u, w_1 and w_2 for a generic finite element of length c (matrices are denoted by underscoring ~):

$$u = \underset{\sim}{\phi}\,\underset{\sim}{u}^{n} \tag{62}$$

$$w_1 = \underset{\sim}{\phi}\,\underset{\sim}{w}_1^{n} \tag{63}$$

$$w_2 = \underset{\sim}{\phi}\,\underset{\sim}{w}_2^{n} \tag{64}$$

where

$$\underset{\sim}{\phi} = \{1 - 3\xi^2 + 2\xi^3 \quad c(\xi - 2\xi^2 + \xi^3) \quad 3\xi^2 - 2\xi^3 \quad c(-\xi^2 + \xi^3)\} \tag{65}$$

is the shape function matrix, dependent on the non-dimensional coordinate

$$\xi = z/c \tag{66}$$

and

$$\underset{\sim}{u}^{nt} = \{u_i \; u_i' \; u_j \; u_j'\} \tag{67}$$

$$\underset{\sim}{w}_1^{nt} = \{w_{1i} \; w_{1i}' \; w_{1j} \; w_{1j}'\} \tag{68}$$

$$\underset{\sim}{w}_2^{nt} = \{w_{2i} \; w_{2i}' \; w_{2j} \; w_{2j}'\} \tag{69}$$

are the corresponding vectors of nodal values.

Introducing these shape functions in the functional P_{ww} (in matrix notation, exponent t stands for the transposed matrix) there results

$$P_{ww} = \frac{1}{2}\int_0^l (u''^t j_g u'' + u'^t SL^2 u' - 2SLu'^t w_1 + 2SLu'^t w_2 - 2u^t p_{ww} + \\ + w_1^t S w_1 - 2Sw_1^t w_2 + w_2^t S w_2 + w_1'^t F_1 w_1' + w_2'^t F_2 w_2')dz \quad (70)$$

or

$$P_{ww} = \frac{1}{2}\int_0^l \Big\{ \underset{\sim}{u}^{nt}\left(j_g \underset{\sim}{\phi}''^t \underset{\sim}{\phi}'' + SL^2 \underset{\sim}{\phi}'^t \underset{\sim}{\phi}'\right)\underset{\sim}{u}^n - \underset{\sim}{u}^{nt}\left(2SL\underset{\sim}{\phi}'^t \underset{\sim}{\phi}\underset{\sim 1}{w}^n + 2SL\underset{\sim}{\phi}'^t \underset{\sim}{\phi}\underset{\sim 2}{w}^n\right) + \\ - 2\underset{\sim}{u}^{nt}\underset{\sim}{\phi}^t p_{ww} + \underset{\sim 1}{w}^{nt}\underset{\sim}{\phi}'^t F_1 \underset{\sim}{\phi}'\underset{\sim 1}{w}^n - \underset{\sim 1}{w}^{nt} 2S\underset{\sim}{\phi}^t\underset{\sim}{\phi}\underset{\sim 2}{w}^n + \underset{\sim 1}{w}^{nt}\underset{\sim}{\phi}^t S\underset{\sim}{\phi}\underset{\sim 1}{w}^n + \\ + \underset{\sim 2}{w}^{nt}\underset{\sim}{\phi}'^t F_2 \underset{\sim}{\phi}'\underset{\sim 1}{w}^n + \underset{\sim 2}{w}^{nt}\underset{\sim}{\phi}^t S\underset{\sim}{\phi}\underset{\sim 2}{w}^n \Big\} c\,d\xi \quad (71)$$

Evaluating the functional first variation:

$$\delta P_{ww} = \delta\underset{\sim}{u}^{nt}\int_0^l\left[\left(j_g\underset{\sim}{\phi}''^t\underset{\sim}{\phi}'' + SL^2\underset{\sim}{\phi}'^t\underset{\sim}{\phi}'\right)\underset{\sim}{u}^n - SL\underset{\sim}{\phi}'^t\underset{\sim}{\phi}\underset{\sim 1}{w}^n + SL\underset{\sim}{\phi}'^t\underset{\sim}{\phi}\underset{\sim 2}{w}^n - \underset{\sim}{\phi}^t p_{ww}\right]c\,d\xi + \\ + \delta\underset{\sim 1}{w}^{nt}\int_0^l\left[-SL\underset{\sim}{\phi}^t\underset{\sim}{\phi}'\underset{\sim}{u}^n + \underset{\sim}{\phi}'^t F_1\underset{\sim}{\phi}'\underset{\sim 1}{w}^n - S\underset{\sim}{\phi}^t\underset{\sim}{\phi}\underset{\sim 2}{w}^n + \underset{\sim}{\phi}^t S\underset{\sim}{\phi}\underset{\sim 1}{w}^n\right]c\,d\xi + \\ + \delta\underset{\sim 2}{w}^{nt}\int_0^l\left[SL\underset{\sim}{\phi}^t\underset{\sim}{\phi}'\underset{\sim}{u}^n - S\underset{\sim}{\phi}^t\underset{\sim}{\phi}\underset{\sim 1}{w}^n + \underset{\sim}{\phi}^{t\prime} F_2\underset{\sim}{\phi}'\underset{\sim 2}{w}^n + \underset{\sim}{\phi}^t S\underset{\sim}{\phi}\underset{\sim 2}{w}^n\right]c\,d\xi \quad (72)$$

Applying the minimum potential energy principle, i.e. imposing that

$$\delta P_{ww} = 0 \quad (73)$$

the following results:

$$\begin{bmatrix} \underset{\sim J}{k} + \underset{\sim S}{k} & -\underset{\sim D}{k} & -\underset{\sim D}{k} \\ -\underset{\sim D}{k}^t & \underset{\sim}{k}_{F1} + \underset{\sim}{k}_B & -\underset{\sim}{k}_B \\ \underset{\sim}{k}_D^t & -\underset{\sim}{k}_B^t & \underset{\sim}{k}_{F2} + \underset{\sim}{k}_B \end{bmatrix} \begin{Bmatrix} \underset{\sim}{u}^n \\ \underset{\sim 1}{w}^n \\ \underset{\sim 2}{w}^n \end{Bmatrix} = \begin{Bmatrix} \underset{\sim}{p} \\ \underset{\sim}{0} \\ \underset{\sim}{0} \end{Bmatrix} \quad (74)$$

where

$$\underset{\sim}{k}_J = \int_0^l j_g \underset{\sim}{\phi}''^t \underset{\sim}{\phi}'' c\,d\xi = \frac{j_g}{c^3}\begin{bmatrix} 12 & 6c & -12 & 6c \\ & 4c^2 & -6c & 2c^2 \\ & & 12 & -6c \\ & & & 4c^2 \end{bmatrix} \text{ symmetric} \quad (75)$$

$$\underset{\sim S}{k} = \int_0^l SL^2 \underset{\sim}{\phi}'^t \underset{\sim}{\phi}' c\,d\xi = \frac{SL^2}{30c}\begin{bmatrix} 36 & 3c & -36 & 3c \\ & 4c^2 & -3c & -c^2 \\ & & 36 & -3c \\ & & & 4c^2 \end{bmatrix} \text{ symmetric} \quad (76)$$

$$\underset{\sim}{k}_D = \int_0^1 SL \underset{\sim}{\phi}'^t \underset{\sim}{\phi} c d\xi = \frac{SL}{60}\begin{bmatrix} -30 & -6c & -30 & 6c \\ 6c & 0 & -6c & c^2 \\ 30 & 6c & 30 & -6c \\ -6c & -c^2 & 6c & 0 \end{bmatrix} \quad \text{anti-symmetric} \tag{77}$$

$$\underset{\sim}{k}_{Fi} = \int_0^1 \underset{\sim}{\phi}'^t F_i \underset{\sim}{\phi}' c d\xi = \frac{F_i}{30c}\begin{bmatrix} 36 & 3c & -36 & 3c \\ & 4c^2 & -3c & -c^2 \\ & & 36 & -3c \\ & & & 4c^2 \end{bmatrix} \quad \text{symmetric} \tag{78}$$

$$\underset{\sim}{k}_B = \int_0^1 S \underset{\sim}{\phi}^t \underset{\sim}{\phi} c d\xi = \frac{Sc}{210}\begin{bmatrix} 78 & 11c & 27 & -6{,}5c \\ & 2c^2 & 6{,}5c & -1{,}5c \\ & & 78 & -11c \\ & & & 2c^2 \end{bmatrix} \quad \text{symmetric} \tag{79}$$

$$\underset{\sim}{p} = \int_0^1 \underset{\sim}{\phi}^t p_{ww} c d\xi \tag{80}$$

Essential boundary conditions may be applied to Equation (74) by using computational procedures that modify the global stiffness matrix, for instance, by adding large numbers to the corresponding diagonal elements. Note that, when using the finite element displacement method, natural boundary conditions need not be imposed. Solving the system of algebraic equations, displacements are obtained, and then strains, stresses, as well as normal and shear forces, together with bending moments, may be calculated.

7. EXAMPLE

Consider the case of a symmetric building whose structure is composed by s parallel shear walls as well as f parallel frames. This may be the case for a building with a typical floor plane where the dimension parallel to the shear walls and the frames is much smaller than that of the orthogonal dimension. Such a case can be analysed by considering one resultant shear wall and one resultant frame connected by pinned bars (representing the action of the floor slabs), as is the case of the application example of the plane panel of Figure 9 (Battistelle, 1991) where one shear wall (0·20 m × 1·50 m) and one single bay frame are connected by pinned bars, and acted upon by a uniform horizontal external load p = 4 kN/m. Column and beam rectangular cross-section dimensions are, respectively, 0·4 m × 0·4 m and 0·2 m × 0·4 m. Storey and building heights are, respectively, 3 m and 60 m. Material Young's modulus is $E = 2 \times 10^7 \text{kN/m}^2$.

Table 1 presents horizontal displacement results obtained by Battistelle (1991), by solving the differential equation (CMT) and using the matrix discrete method (MDM) computer program; it also shows the results obtained here by the use of the one-dimensional finite element method (FEM) computer program.

The total number of finite elements was made equal to 20, thus having individual length equal to the storey height, that is, h = 3·0 m. The finite element computer program had been earlier prepared to deal only with the so-called basic case of two shear walls associated by lintel beams (functional (49)), but according to the unified approach presented in this paper it was also possible to use it to solve the structural problem proposed in the above example! This was possible because expressions (52) were used to define the shear wall frame structure, equivalent to the shear walls associated by

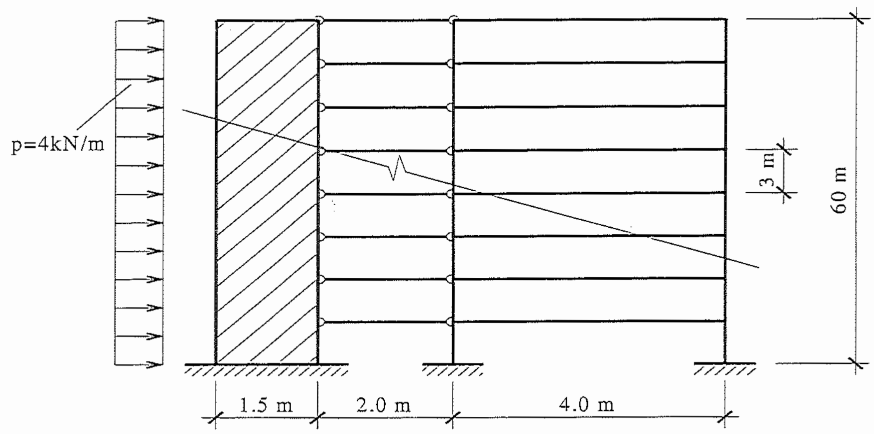

Figure 9. Frame–shear wall association by pinned bars

Table 1. Panel horizontal displacements

Storey level (m)	Horizontal displacements u(z) (m)		
z	CMT	MDM	FEM
0·0	0·0000	0·0000	0·0000
3·0	0·0065	0·0064	0·0065
6·0	0·0232	0·0229	0·0232
9·0	0·0467	0·0462	0·0467
12·0	0·0745	0·0740	0·0746
15·0	0·1052	0·1047	0·1053
18·0	0·1375	0·1371	0·1376
21·0	0·1705	0·1704	0·1706
24·0	0·2037	0·2039	0·2039
27·0	0·2366	0·2372	0·2368
30·0	0·2688	0·2699	0·2691
33·0	0·3002	0·3018	0·3005
36·0	0·3305	0·3326	0·3309
39·0	0·3596	0·3622	0·3600
42·0	0·3873	0·3905	0·3877
45·0	0·4136	0·4174	0·4141
48·0	0·4386	0·4429	0·4392
51·0	0·4623	0·4671	0·4629
54·0	0·4848	0·4899	0·4854
57·0	0·5063	0·5119	0·5070
60·0	0·5273	0·5330	0·5280

lintel beams. Very few parameters are needed to define the structure; thus the data file is very simple. The computer solution is obtained almost instantaneously.

8. CONCLUSION

Table 1 shows that a very good agreement is achieved when comparing CMT to FEM procedures, both based on the same assumptions. On the other hand, the classical MDM procedure confirms the validity of our proposal related to the tall building structures unified plane panels formulation, also presented earlier in Mancini and Savassi (1999), through its local formulation. As also stated in that earlier paper, for cases where geometry or loading variations occur, the approach presented in this paper is even more powerful because it will always give a very good numerical answer for the problem, which certainly would be much more difficult to solve by the traditional analytical formulation. Examples of such irregular structures may be found in Savassi (1993) (no axial deformations were taken in this case) or Savassi (1996).

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NOTATION

A_i, A_{pi}	i^{th} shear wall or column cross-sectional area
b_i	Auxiliary value
E	Material longitudinal modulus of elasticity
f	Final distance between two shear wall cross-sections, under bending and axial actions
F_1, F_2	Values of EA_1 and EA_2
P_f	Frame total potential energy (functional)
P_w	Wall total potential energy
P_{fw}	Frame shear wall total potential energy
P_{ww}	Shear walls association total potential energy
h	Constant storey height (distance between floors)
H	Total height of the building
I_c	Lintel (connecting) beam second moment of area
I_{wi}	i^{th} column or shear wall second moment of area
I_w	Single shear wall second moment of area
j_i	i^{th} column or shear wall bending rigidity product EI_i
$j_g = E(I_{w1} + I_{w2})$	Assembled shear walls rigidity product
j_w	Single shear wall rigidity product EI_w
k_i, k_c	i^{th} column rigidity quotient I_i/h
k_v	Frame beam rigidity quotient I_v/ℓ_v
ℓ_v, L	Distance between column or shear wall vertical centroidal axes
M	External forces bending moment
M_i	Shear wall bending moment
M_{AC}	Node A bending moment due to AC bar action
N_i	Column or shear wall axial force
p	Distributed external horizontal force
p_r	Distributed horizontal force received by the frame
p_w	Distributed horizontal force received by the shear wall
u, (*v*), w	x, (y) and z linear displacement components
q_c	Diaphragm distributed shear force

V	External loading shear force
V_i	i^{th} column shear force
V_f	External loading frame shear force
Q_c	Beam shear force
x, y, z	Cartesian coordinates. Horizontal x and vertical z axes define panel plane
w_1, w_2	Column axial displacements
β	Frame node rotation
$\approx$	Indicates likeness between two functionals
S	Lintel bending rigidity
ℓ	Lintel beam length

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ANÁLISE DO COMPORTAMENTO ESTÁTICO DE PAINEIS PLANOS DE EDIFÍCIOS ALTOS UTILIZANDO A TÉCNICA CONTÍNUA

Maria Ângela P. Xavier
Departamento de Tecnologia
Centro Universitário de João Pessoa – UNIPÊ
BR 230 – Km 22 – 58.053.000 – João Pessoa – PB – Brazil
mangelaxavier@uol.com.br
Eddie Mancini
Departamento de Estruturas
Escola de Engenharia de São Paulo – USP
Av. do Trabalhador Sãocarlense, 400 – 13566590 – São Carlos SP - Brazil

Resumo. *Neste trabalho apresenta-se um processo de análise em regime elástico, em teoria de primeira ou segunda ordens, para painéis planos de edifícios altos, sujeitos a ação de cargas laterais e verticais, uniformemente distribuídas ao longo da altura da estrutura. Utiliza-se para o desenvolvimento teórico a técnica do meio continuo, que consiste basicamente em considerar a rigidez dos elementos de conexão horizontais, lajes e vigas, distribuída na altura da estrutura. Considera-se na análise: a influência das deformações axiais dos pilares; a influência da deformação por força cortante das barras e, também, a influência dos trechos rígidos, que são as regiões de interseção entre vigas e pilares. Admite-se que a flexão total nos elementos verticais, a nível dos andares, é obtida pela soma da flexão local (correspondente a igualdade das rotações de três nós consecutivos) e da flexão global (correspondente à variação da rotação dos nós com a altura). Com o modelo estrutural assim proposto, é possível analisar, pela técnica do meio contínuo, qualquer painel plano com prumadas verticais de pilares parede e ou pilares de pórtico, sem que seja necessário o enquadramento prévio de cada prumada vetical nas categorias citadas.*

Palavras-chave: *Técnica do meio contínuo, Painéis planos de edifícios altos, Efeitos de segunda ordem*

1. INTRODUÇÃO

Os processos de cálculo utilizando a técnica do meio contínuo exigiam, até então, para a elaboração do modelo estrutural e a análise respectiva, o enquadramento prévio de cada prumada vertical nas categorias de pilar parede ou pilar de pórtico, exigindo portanto para cada painel uma modelagem particular.

Este trabalho apresenta um processo de análise desenvolvido em Xavier (1994). Este processo permite uma análise elástica em teorias de primeira ou segunda ordens, utilizando a técnica contínua, de painéis planos de edifícios altos. Consideram-se a influência das deformações axiais dos pilares, conforme Battistelle (1991), a influência da deformação por força cortante das barras e também a influência dos trechos rígidos, regiões de interseção entre vigas e pilares, conforme Fakury (1992). Segundo Laier (1989), em cada elemento vertical do painel é considerada a flexão local à qual, em cada andar, é acrescentada a flexão global do elemento considerado como um todo. A flexão global predomina nos pilares parede, já a flexão local predomina nos pilares do pórtico. No processo de análise em estudo este comportamento à flexão dos elementos verticais é caracterizado naturalmente. Isto permite que o modelo estrutural proposto possa ser utilizado para a análise de qualquer painel plano, pela técnica do meio contínuo. Assim este modelo pode tratar painéis com prumadas verticais de pilares parede e ou pilares de pórtico, sem qualquer limitação relativa a seu posicionamento na estrutura.

O comportamento do painel considerado fica expresso através de um sistema de equações diferenciais de terceira ordem. A solução deste sistema de equações é obtida através do processo das diferenças finitas, com uso de computador, e fornece os deslocamentos laterais do painel e os deslocamentos verticais de cada prumada de pilar ao longo da altura.

2. EQUAÇÕES DE EQUILIBRIO

2.1. Considerações iniciais

Na Fig.1 representa-se o painel plano em sua posição deformada. Este painel é constituído por n pilares, de seções transversais não necessariamente iguais, e m $(m=n-1)$ vigas, com vãos de comprimentos D_k (o índice k indica a numeração dos vãos). D representa a largura do painel, H sua altura, e h a altura dos andares. O carregamento externo é constituído por: carga lateral uniformemente distribuída q; carga lateral concentrada no topo F e carga vertical uniforme p, distribuída entre os pilares ao longo de seus eixos verticais.

Admite-se para o material o comportamento elástico linear, representando-se por E e G os módulos de elasticidade longitudinal e transversal, respectivamente.

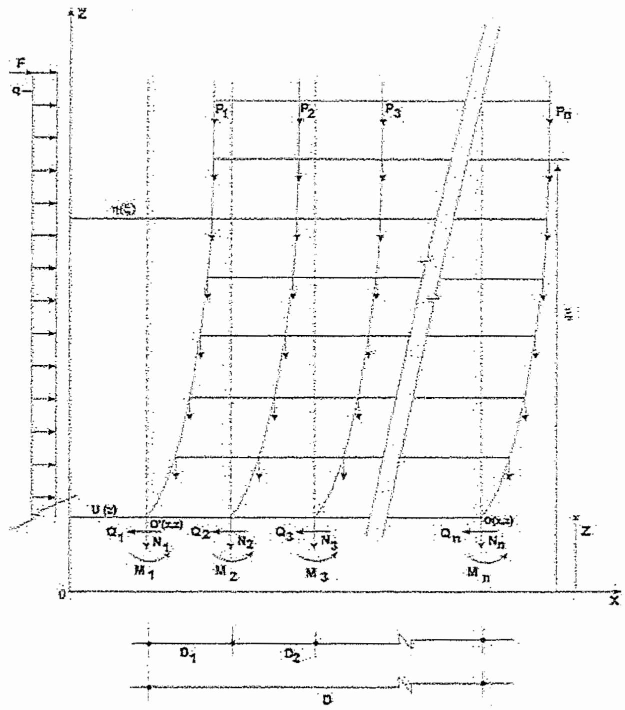

Figura 1 – Painel plano em sua posição deformada

2.2. Equilíbrio do painel à força normal

Indica-se na Fig.2 um elemento genérico do pilar *j*. p_j indica a carga vertical e uniforme, N_j a força normal e q_k, de acordo com a técnica contínua, é a força cortante distribuída no meio contínuo que constitui o sistema discreto de vigas do vão *k*.

Considerando-se na Fig.2 positivos os esforços de tração, o equilíbrio à força normal, respectivamente para o pilar da extremidade esquerda, para o pilar genérico *j* e para o pilar da extremidade direita, será expresso por:

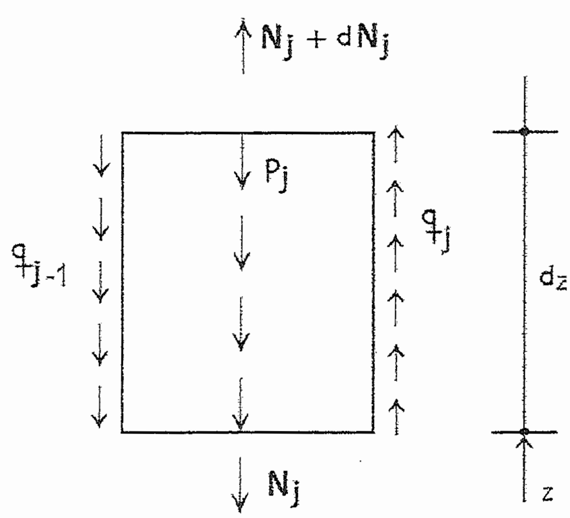

Figura 2 – Elemento do pilar genérico *j*

$$N'_1 = E\,Ap_1 v_1'' = p_1 - q_1$$
$$N'_j = E\,Ap_j v_j'' = p_j + q_{j-1} - q_j$$
$$N'_n = E\,Ap_n v_n'' = p_n + q_{n-1} \qquad (1)$$

onde Ap_j é a área de seção transversal e v''_j é a derivada segunda, em relação a z, do deslocamento vertical v_j do pilar j.

2.3. Equilíbrio do painel ao momento

Considerando-se positivos os sentidos para os esforços e deslocamentos indicados na Fig.1, o equilíbrio ao momento em relação ao ponto $o(x,z)$, de todos os esforços externos e internos, expressa-se por:

$$N_1 D + N_2 (D - D_1) + N_3 (D - D_1 - D_2) + \ldots + N_{n-1} D_{n-1} + (M_1 + M_2 + M_3 + \ldots + M_n) = M_\ell - \int_z^H p_1 [D - (\eta - u)]\, d\xi - \int_z^H p_2 [D - D_1 - (\eta - u)]\, d\xi - \int_z^H p_3 [(D - D_1 - D_2) - (\eta - u)]\, d\xi - \ldots - \int_z^H p_{n-1} [D_{n-1} - (\eta - u)]\, d\xi + \int_z^H p_n (\eta - u)\, d\xi \qquad (2)$$

onde M_ℓ é o momento, em cada cota z, das cargas laterais q e F e u e $\eta(\xi)$ são os deslocamentos laterais nas cotas z e ξ, respectivamente.

Efetuando-se as integrais do segundo membro da Eq.(2), derivando-se uma vez, em relação a z, a equação resultante, e considerando-se, conforme Fig.1, que na cota z $\eta(z) = u$, obtém-se, tendo-se também em vista as Eqs.(1), a seguinte equação de equilíbrio à força cortante

$$Q_\ell = \sum_{k=1}^{n-1} q_k D_k - \sum_{j=1}^{n} M'_j - p\,(H - z)\, u' \qquad (3)$$

onde Q_ℓ é a força cortante externa, em uma cota z, devido às cargas laterais q e F e M'_j é a derivada primeira do momento fletor M_j.

Considera-se, inicialmente, que sob a ação das cargas externas, o andar genérico i do painel apresenta a deformação representada na Fig.3. Em virtude da distorção u' do painel e das rotações dos nós, os pilares apresentam neste nível uma flexão local expressa pelos momentos $M'_{j,i}$ e $M''_{j,i+1}$. A estes momentos deve-se superpor a flexão global, expressa pelos momentos M_j.

De acordo com a Fig.3 definem-se: a_j como a rotação do nó A_j do pilar j; ψ_k como a inclinação da viga k (devido aos deslocamentos verticais dos pilares k e $k+1$) e ϕ_k^1 e ϕ_k^2 como as rotações das extremidades esquerda e direita, respectivamente, da viga situada no vão k.

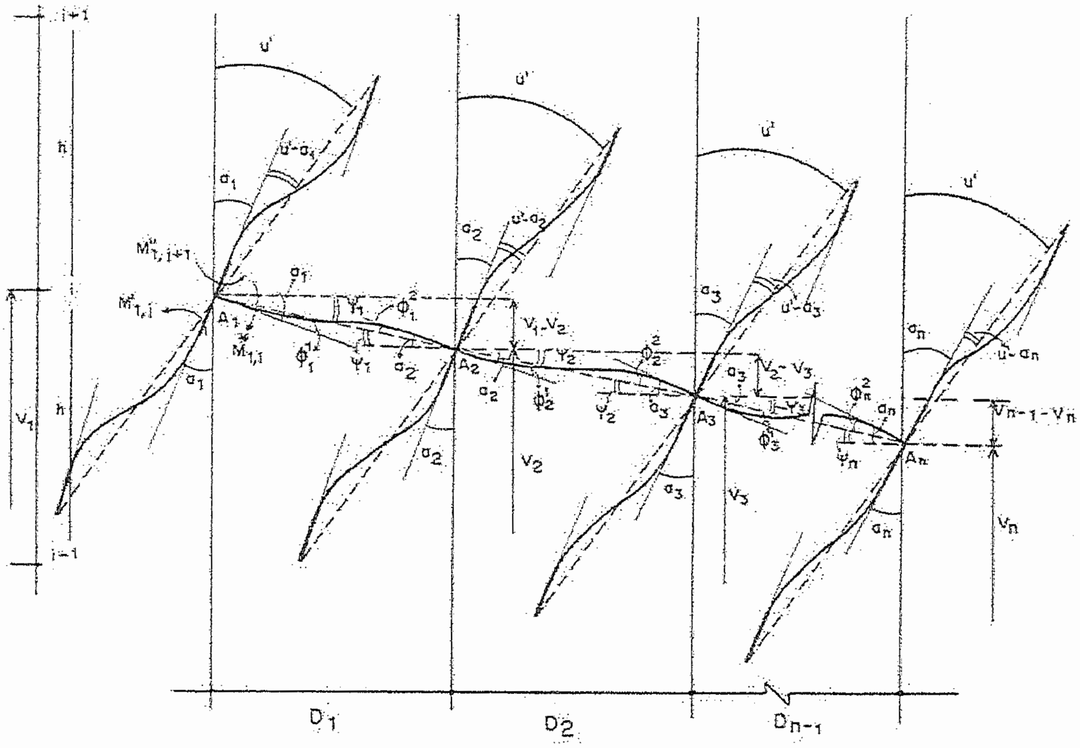

Figura 3 – Trecho de um painel e sua deformação devido ao carregamento

A viga genérica *k* estará solicitada por uma força cortante concentrada Vv_k ($Vv_k = h.q_k$, sendo *h* a altura dos andares) e pelos momentos $M^{*}_{k,k}$ e $M^{**}_{k+1,k}$, gerados pela rotações dos nós e deslocamentos verticais dos pilares *k e k+1*. Pode-se então escrever:

$$q_k \, D_k = \frac{M^{*}_{k,k} + M^{**}_{k+1,k}}{h} \qquad (4)$$

Neste trabalho, considerar-se-á como dimensão horizontal do trecho rígido a largura do pilar e como dimensão vertical, a altura da viga. Na Fig.4 representam-se estas dimensões, definidas da seguinte maneira: Cv_k é o comprimento do trecho deformável da viga do vão *k*; $\bar{h}$ é o comprimento do trecho deformável dos pilares do andar *i;* tv_k é a semi-largura do trecho rígido do pilar *j;* tv_{k+1} é a semi-largura do trecho rígido do pilar *j+1;* t_j é a semi-altura do trecho rígido das vigas dos vãos *k* e *k –1* que concorrem no pilar *j*. No caso de alturas diferentes, considera-se a altura da viga mais alta.

Denominar-se-ão nós puntuais aos pontos de interseção dos eixos longitudinais dos pilares e vigas.

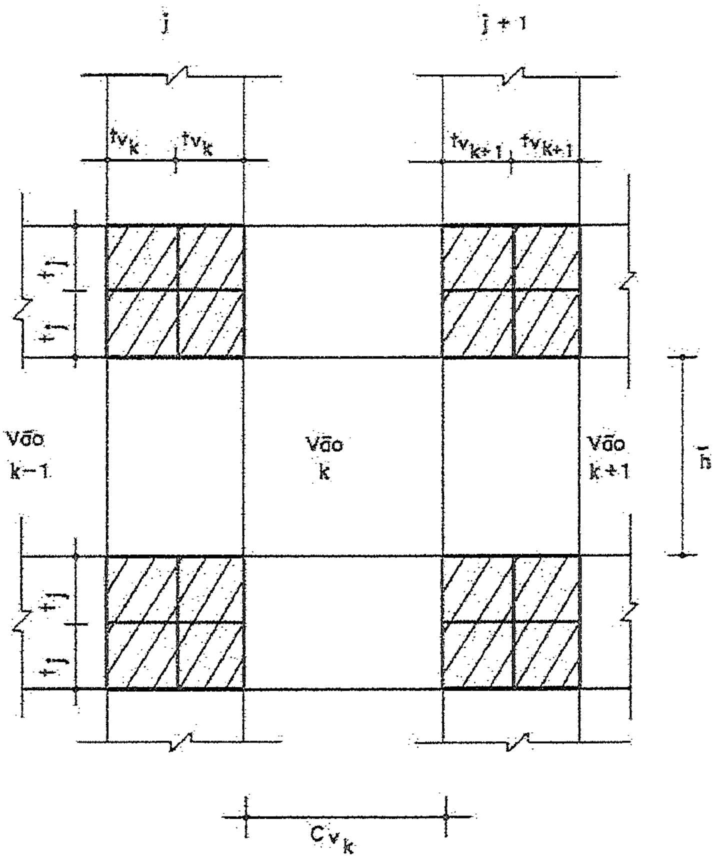

Figura 4 – Representação dos trechos rígidos do painel

Os momentos $M^{*}_{k,k}$ e $M^{**}_{k+1,k}$ valem:

$$M^{*}_{k,k} = \frac{6.E.Ivc_k}{D_k} \left[2\gamma_k \phi^1_k + \mu_k \phi^2_k \right]$$

$$M^{**}_{k+1,k} = \frac{6.E.Ivc_k}{D_k} \left[\bar{\mu}_k \phi^1_k + 2\bar{\gamma}_k \phi^2_k \right] \tag{5}$$

onde Ivc_k é o momento de inércia corrigido da viga, que leva em conta a deformação por força cortante, e que vale: $Ivc_k = rv_k \, . Iv_k$, com:

$$rv_k = \frac{Av_k \, .G.Cv^2_k}{Av_k .G.Cv^2_k + 12.c.E.Iv_k} \tag{6}$$

onde Av_k é a área da seção transversal e Iv_k é o momento de inércia da viga do vão k e c é o fator de forma da seção transversal do trecho deformável da viga.

Os parâmetros $\gamma_k, \bar{\gamma}_k, \mu_k, \bar{\mu}_k$ que aparecem nas Eqs.(5) são calculados através das expressões:

$$\gamma_k = \left(\frac{D_k}{Cv_k}\right)^3 \left[\frac{Cv_k}{2rv_k}\,(tv_k + (\,tv_k - sv_k)\,rv_k) + \frac{Cv_k^2}{3rv_k} + tv_k\,(tv_k - sv_k)\right]\frac{1}{D_k^2}$$

$$\bar{\gamma}_k = \left(\frac{D_k}{Cv_k}\right)^3 \left[\frac{Cv_k}{2rv_k}\,(tv_{k+1} + (tv_{k+1} - sv_k)\,rv_k) + \frac{Cv_k^2}{3rv_k} + tv_{k+1}\,(tv_{k+1} - sv_k)\right]\frac{1}{D_k^2}$$

$$\mu_k = \left(\frac{D_k}{Cv_k}\right)^3 \left[\frac{Cv_k}{rv_k}\,((tv_k - sv_k)\,rv_k + tv_{k+1}) + \frac{Cv_k^2}{3rv_k} + 2tv_{k+1}\,(tv_k - sv_k)\right]\frac{1}{D_k^2}$$

$$\bar{\mu}_k = \left(\frac{D_k}{Cv_k}\right)^3 \left[\frac{Cv_k}{rv_k}\,(tv_k + (tv_{k+1} - sv_k)\,rv_k) + \frac{Cv_k^2}{3rv_k} + 2tv_k\,(tv_{k+1} - sv_k)\right]\frac{1}{D_k^2} \qquad (7)$$

onde :

$$sv_k = \frac{6.c.E.Iv_k}{Av_k.G.Cv_k} \qquad (8)$$

Os valores dos ângulos ψ_k, ϕ_k^1 e ϕ_k^2 são facilmente obtidos na Fig.3. Levando estes valores nas Eqs.(5) e substituindo-se as equações resultantes na Eq.(4), obtém-se:

$$q_k\,D_k = \frac{6.E.Kvc_k}{h}\Big[\,(\bar{\mu}_k + 2\gamma_k)\,a_k + (\mu_k + 2\bar{\gamma}_k)\,a_{k+1} + $$
$$- (\mu_k + \bar{\mu}_k + 2\gamma_k + 2\bar{\gamma}_k)\,\frac{v_k - v_{k+1}}{D_k}\Big] \qquad (9)$$

com Kvc_k sendo o parâmetro de rigidez da viga dado por: $Kvc_k = Ivc_k \,/\, D_k$

A equação acima desdobra-se em $(n-1)$ equações relativas aos $n-1$ vãos. Em forma matricial tem-se:

$$\sum_{k=1}^{n-1} q_k\,D_k = \{E\}([KB]\{A\} - [KBB]\{v\}) \qquad (10)$$

O vetor $\{E\}$ é de ordem (1xn) com todos os elementos de valor $6E/h$. Ao nível do andar i, $\{A\}$ é o vetor das rotações dos nós e $\{v\}$ é o vetor dos deslocamentos verticais dos pilares. $[KB]$ e $[KBB]$ são matrizes quadradas de ordem n.

Impondo-se a condição de equilíbrio ao momento em cada nó A_j do andar i, é possível expressar o vetor $\{A\}$ como uma função dos deslocamentos laterais u e dos deslocamentos verticais v_j. De acordo com a Fig.3, tem-se, respectivamente para o nó da extremidade esquerda, para um nó intermediário e para o nó da extremidade direita, as seguintes expressões:

$$M''_{1,i+1} + M'_{1,i} - M^*_{1,1} = 0$$

$$M''_{j,i+1} + M'_{j,i} - M^{**}_{j,j-1} - M^{*}_{j,j} = 0$$
$$M''_{n,i+1} + M'_{n,i} - M^{**}_{n,n-1} = 0 \quad (11)$$

Supondo-se que são próximos os valores das rotações de três nós consecutivos de um mesmo pilar, os momentos $M''_{j,i+1}$ e $M'_{j,i}$ valem, ambos, $6.E.Ipc_j\,.\,\rho_j\,(u' - a_j)\,/\,h$. Ipc_j é o momento de inércia corrigido do pilar j, que leva em conta a deformação por força cortante, e que vale: $Ipc_j = rp_j\,.\,Ip_j$, onde:

$$rp_j = \frac{Ap_j\,.\,G.\bar{h}^2}{Ap_j\,.\,G.\bar{h}^2 + 12E.c.Ip_j} \quad (12)$$

com Ip_j sendo o momento de inércia, Ap_j a área da seção transversal, $\bar{h}$ o comprimento do trecho deformável e c o fator de forma do trecho deformável do pilar j. Sendo $sp_j = 6.c.E.Ip_j/(Ap_j\,G.\bar{h})$, define-se ρ_j como:

$$\rho_j = \left(\frac{h}{\bar{h}}\right)^3 \left[\frac{\bar{h} + 2\,(t_j - sp_j)\,rp_j}{h.rp_j}\right] \quad (13)$$

$M^{*}_{j,j}\,(j = 1, 2, \ldots, n-1)$ e $M^{**}_{j,j-1}\,(j = 2, 3 \ldots, n)$ são, respectivamente, os momentos no nó A_j correspondentes às vigas dos vãos j e $j-1$. Estes momentos são dados pelas Eqs.(5), quando se substitui os índices (k, k) por (j, j), na primeira equação, e $(k+1, k)$ por $(j, j-1)$, na segunda equação. Levando-se estes momentos assim obtidos e as expressões para $M''_{j,i+1}$ e $M'_{j,i}$ $(j = 1, 2, \ldots, n)$ nas Eqs.(11), chega-se na seguinte equação matricial:

$$\{A\} = [K_1]\{UN\}\,u' + [K_2]\{v\} \quad (14)$$

onde $[K_1]$ e $[K_2]$ são matrizes quadradas, de ordem n, definidas por: $[K_1] = ([K] + [KB])^{-1}[K]$ e $[K_2] = ([K] + [KB])^{-1}([KBB] + [KBBB])$, com $[K]$ sendo uma matriz diagonal, de ordem nxn, cujos elementos são dados por: $K_j = 2.Ipc_j\,.\rho_j\,/h$ e $[KBBB]$ é uma matriz quadrada de ordem n. O vetor $\{UN\}$ é unitário de ordem $nx1$.

Substituindo-se a Eq.(14) na Eq.(10), obtém-se:

$$\sum_{K=1}^{n-1} q_k\,D_k = s_f u' + \{SB\}\{v\} \quad (15)$$

onde s_f é a rigidez do painel à força cortante de valor $\{E\}[KB][K_1]\{UN\}$ e $\{SB\}$ é um vetor de ordem $1xn$ cuja formação é definida quando se desenvolve a expressão matricial $\{E\}\,([KB]\,[K_2] - [KBB])$

Conforme foi dito anteriormente, o vetor $\{A\}$ fornece ao nível de cada andar as rotações dos pilares cujos valores eram considerados próximos em três nós consecutivos de uma mesma prumada. Isto conduz a um ponto de momento nulo na semi-altura de cada pilar. Considerando-se que, em presença dos momentos fletores M_j, as rotações dos nós das colunas devem variar ao longo da altura e, também, como se utiliza a técnica do meio contínuo, a curvatura das colunas é dada por:

$$\frac{d}{dz}\{A\} = [K_1]\{UN\}\, u'' + [K_2]\{v'\} \tag{16}$$

e a flexão global total, que corresponde à soma dos momentos das colunas, $\sum_{j=1}^{n} M_j$, é expressa por:

$$\sum_{j=1}^{n} M_j = \{E.Ip_1\ E.Ip_2\ ......\ E.Ip_n\}\ \frac{d\{A\}}{dz} \tag{17}$$

Substituindo-se a Eq.(16) na Eq.(17) e levando-se a expressão resultante, derivada uma vez em relação a *z*, juntamente com a Eq.(15) na Eq.(3), obtém-se:

$$Q_l = (s_f - p(H - z))\, u' - K_5 u''' + \{SB\}\{v\} - \{K_6\}\{v''\} \tag{18}$$

onde $K_5 = \{RF\}[K_1]\{UN\}$ e $\{K_6\} = \{RF\}[K_2]$, com $\{RF\}$ sendo um vetor linha de ordem $1xn$ cujos elementos tem valor $E.\ Ip_j\ (j = 1, ... n)$.

3. EQUAÇÃO DIFERENCIAL DO PAINEL

Com a Eq.(18) e as Eqs.(1), de equilíbrio dos pilares à força normal, pode-se compor um sistema de equações em forma matricial, cuja solução fornecerá os deslocamentos laterais $u(z)$ do painel e os deslocamentos verticais $v_j\,(z)$ dos pilares.

Explicitando-se os q_k $(k = 1, ... n - 1)$ na Eq.(9) e levando-se as expressões resultantes nas Eqs.(1) encontra-se a seguinte equação matricial:

$$[E]\{v''\} + \{K_3\}\, u' + [K_4]\{v\} = \{p\} \tag{19}$$

onde: $[E]$ é uma matriz diagonal de ordem nxn cujos elementos tem valor E; $\{K_3\}$ é um vetor de ordem $n \times 1$ definido como: $\{K_3\} = [ES]\,[KBB]^T\,[K_1]\{UN\}$, com $[ES]$ sendo uma matriz diagonal de ordem nxn, cujos elementos tem valor $6.\ E\ /\ (h\ .\ Ap_j)$, $(j = 1, ... n)$; $[K_4]$ é uma matriz quadrada, de ordem n, com sua formação determinada pelo desenvolvimento da expressão: $[K_4] = [ES]\,([KBB]^T\,[K_2] - [KS])$.

As equações matriciais, Eq.(18) e Eq.(19), formam um sistema de $n+1$ equações diferenciais expresso por:

$$[T]\{U''\} + [MS]\{U\} = \{C\} \tag{20}$$

onde: $\{U''\}$ é um vetor de ordem $(n+1)x1$ tendo u''' como primeiro elemento e $\{v''\}_{nx1}$ como os demais elementos; $\{U\}_{(n+1)x1}$ tem u' como primeiro elemento e $\{v\}$ como os demais; $\{C\}_{(n+1)x1}$ é o vetor contendo Q_l como primeiro elemento e o vetor $\{p\}_{nx1}$, das cargas verticais, compondo o conjunto dos demais elementos.

A solução da Eq.(20) será obtida através do método das diferenças finitas. As condições de contorno utilizadas para a resolução do sistema de equação diferenciais serão: $u(0) = 0$ e $\{v\ (0)\} = 0$ (supondo indeslocabilidade dos pilares na base); $u'(0)=0$ (supondo rotação $\{A\}$

nula nas bases dos pilares; $u''(H) = 0$ e $\{v'(H)\} = \{0\}$ (supondo não existir momentos e forças normais aplicadas no topo do painel).

4. OBSERVAÇÕES FINAIS

Conforme foi exposto anteriormente, os pilares apresentam, em cada nível, uma flexão local devido à distorção u' e às rotações dos nós (consideradas próximas em três níveis consecutivos) e também, uma flexão global devido à variação de rotação dos nós ao longo da altura do painel. A flexão total, a nível dos andares, deve ser obtida somando-se as duas flexões citadas. No caso de paredes, as rotações dos nós têm valores aproximadamente iguais ao de u'. Isto equivale a afirmar que, em cada parede, a flexão total, a nível de cada andar, deve ter valor próximo ao da flexão global. No caso de pilares de pórtico isto não ocorre, e a flexão total deve ser obtida pela soma das duas flexões:

5. EXEMPLO

Serão analisadas duas associações de parede e pórtico. A primeira associação ocorre por barras biarticuladas e a segunda, por vigas. As barras biarticuladas têm o comprimento de *2,0m*. As vigas têm também o comprimento de *2,0m* e as seções transversais, constantes ao longo da altura, medem *0,20 x 0,40m*. O painel tem vinte andares de altura $h = 3.0m$. As seções transversais da parede e todos os pilares e vigas do pórtico são constantes ao longo da altura e medem *1.5 x 0.20m* para a parede, *0.4 x 0.4m* para os pilares do pórtico e *0.20 x 0.40m* para o conjunto de vigas. O carregamento constitui-se de: carga lateral $q = 4.0\ kN/m$ e carga vertical $p = 100\ kN/m$ distribuída da seguinte maneira: $p_1 = 48.4\ kN/m$ (parede) e $p_2 = p_3 = 25.8\ kN/m$ (nos pilares do pórtico). Para o módulo de elasticidade adota-se o valor $E = 2 \times 10^7\ kN/m^2$.

Em relação à consideração dos nós rígidos, o painel (pórtico geral) será analisado de duas maneiras:

1º) Associação de parede e pórtico por barras biarticuladas: caso (a) – pórtico de nós pontuais e caso (b) – pórtico de nós rígidos.

2º) Associação de parede e pórtico por vigas: caso (a) – cosiderando-se apenas a dimensão horizontal do nó rígido no primeiro pilar (parede). Os dois pilares restantes (pilares de pórtico) têm nós puntuais e caso (b) – pórtico de nós rígidos.

Os esforços e deslocamentos resultantes da análise em 1ª ordem ($p = 0.0$), são apresentados em gráficos. No caso (a), apresentam-se ainda os resultados obtidos através da análise discreta. Como resultados da análise em 2ª ordem são mostrados apenas os deslocamentos laterais.

Associação de parede e pórtico por barras biarticuladas-resultados:

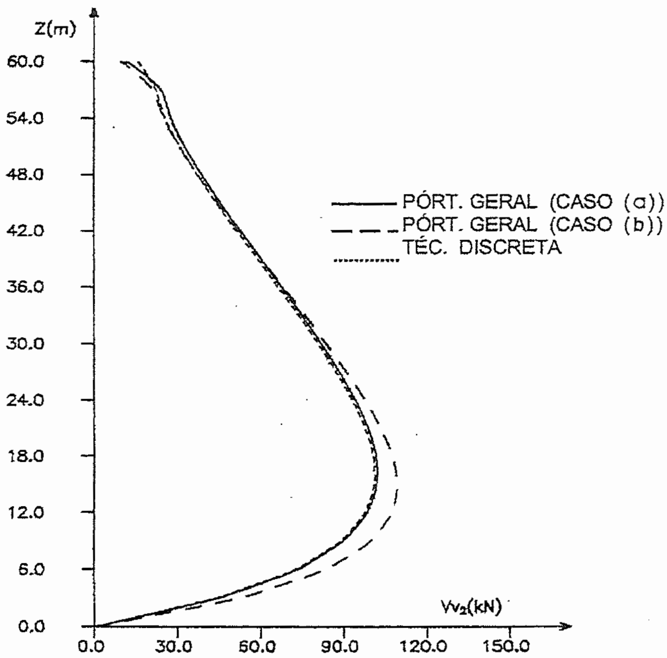

Figura 5 – Força cortante nas vigas do pórtico

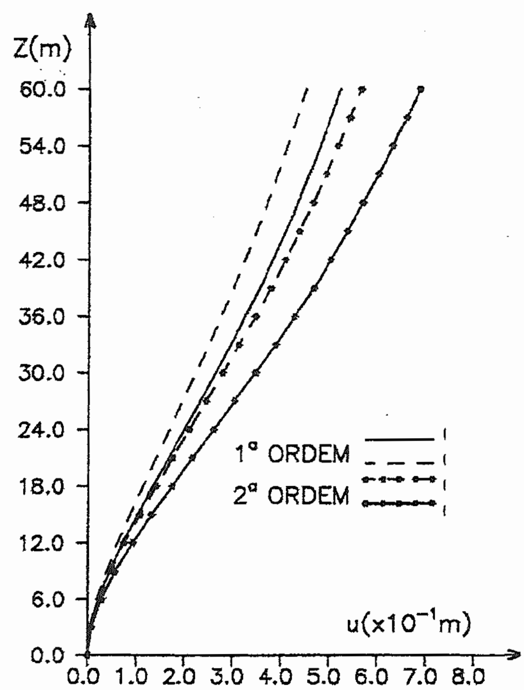

Figura 6 – Deslocamentos laterais u

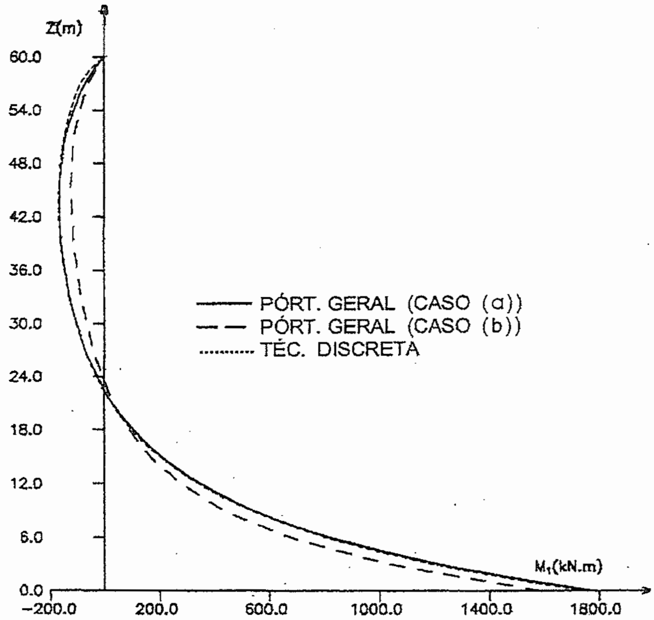

Figura 7 – Momento fletor no pilar 1

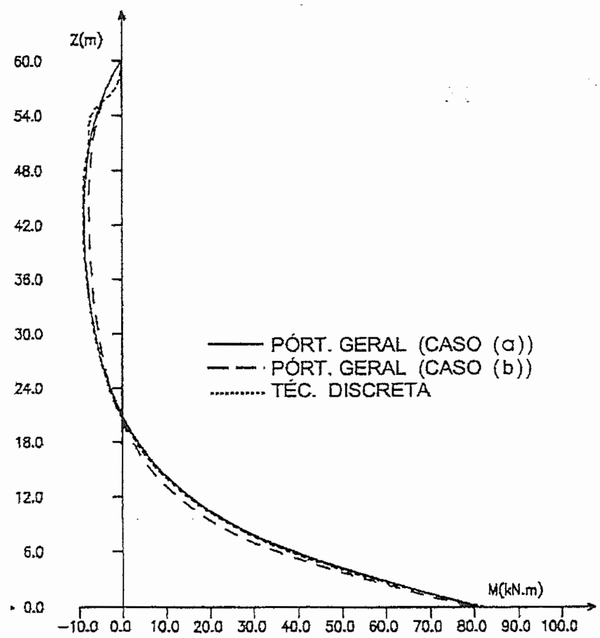

Figura 8 – Momento fletor nos pilares 2 e 3

Associação de parede pórtico por vigas-resultados:

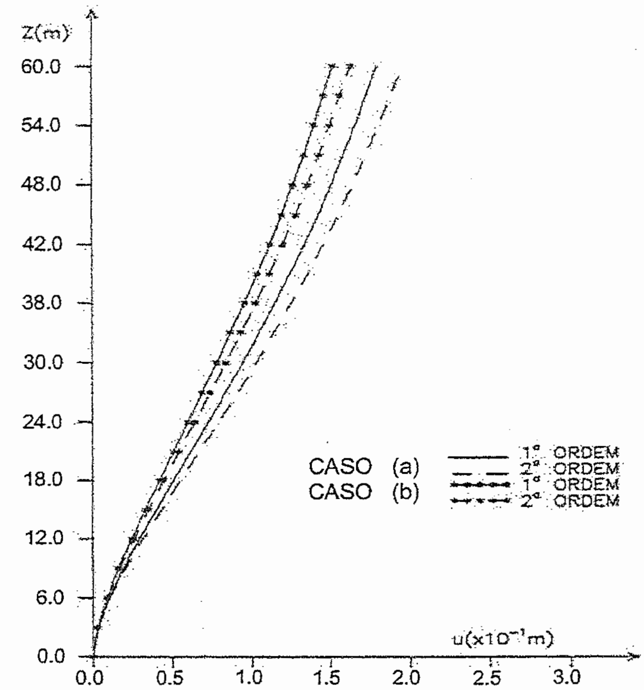

Figura 9 – Deslocamentos laterais u

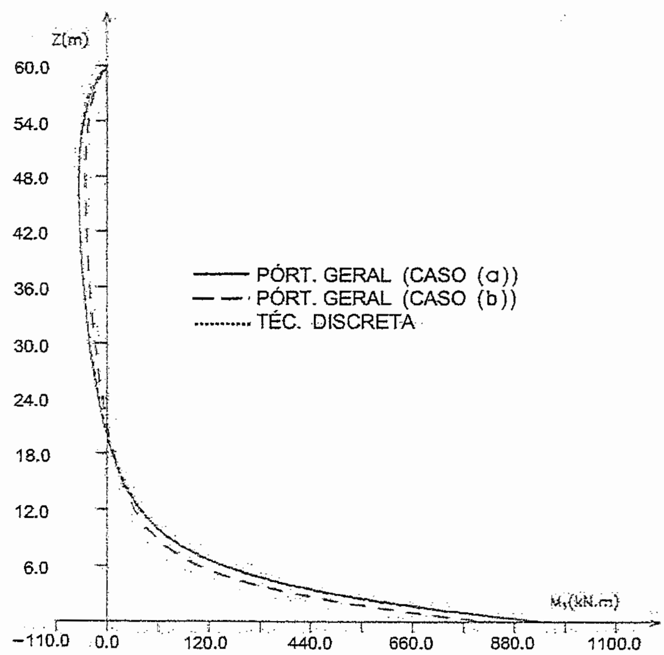

Figura 10 – Momento fletor no pilar 1

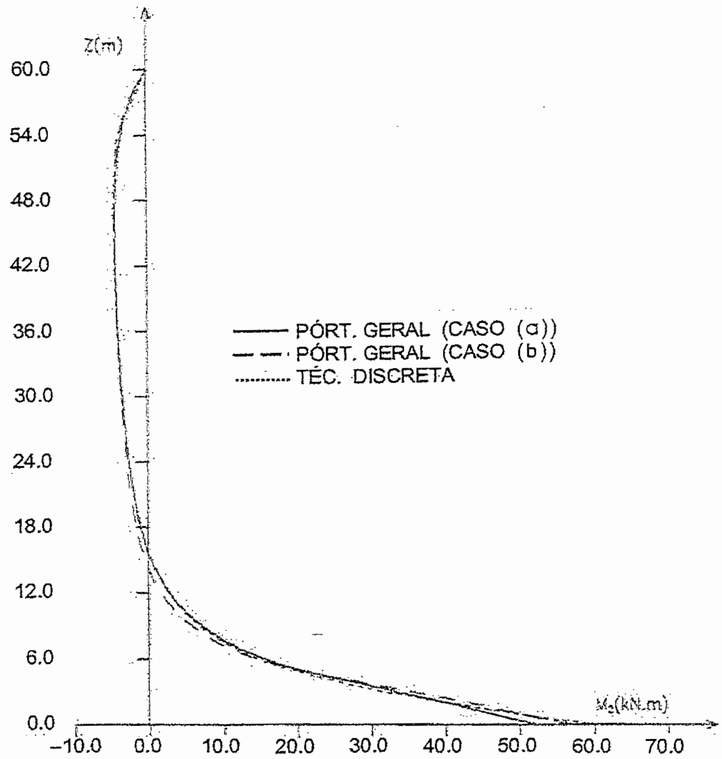

Figura 11 – Momento fletor no pilar 2

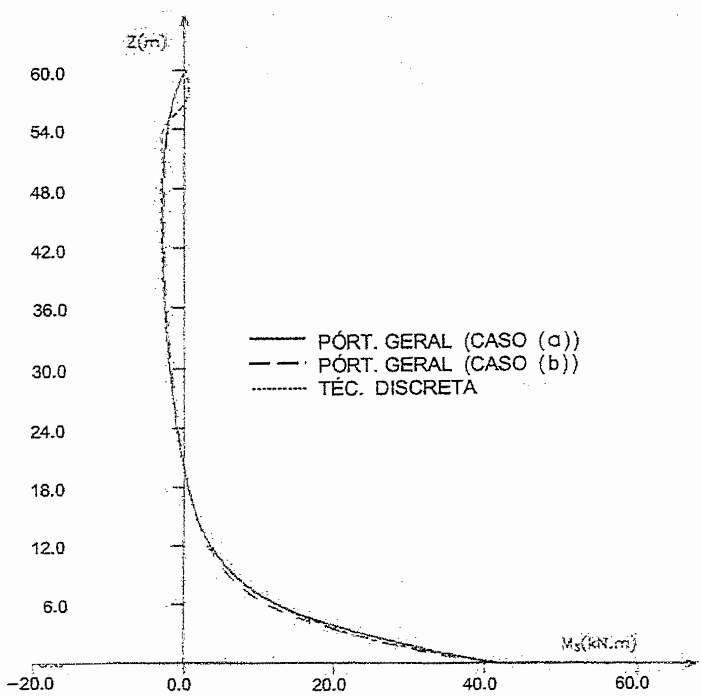

Figura 12 – Momento fletor no pilar 3

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PLANE ASSOCIATION OF BENDING MOMENT AND SHEAR FORCE DEFORMABLE PANELS: SHEAR WALL-FRAME PLANE ASSOCIATION

Eddie Mancini
Walter Savassi
savassi@sc.usp.br
Department of Structural Engineering – São Carlos School of Engineering-USP –
Caixa Postal 359 São Carlos, SP – 13560-970 BRAZIL

Abstract. *This paper presents, by using the continuous medium technique, the formulation for the shear wall-frame plane association supporting lateral loading. Axial deformations of columns are taken into account and the shear wall as well as the frame are both considered as deformable panels if stressed by bending moment and shear force. The proposed procedure is very simple and results obtained, in a direct way, are very close to those obtained by other formulations, as shown by an example.*

Keywords: *Tall buildings, Wind loading, Continuous medium technique, Bracing panels.*

1. INTRODUCTION

It is known that the structural system formed by the plane association of a frame and a shear wall, through extremely rigid horizontal pinned-bars, as shown in Fig. 1, is very efficient to receive horizontal loading due to wind action in tall buildings.

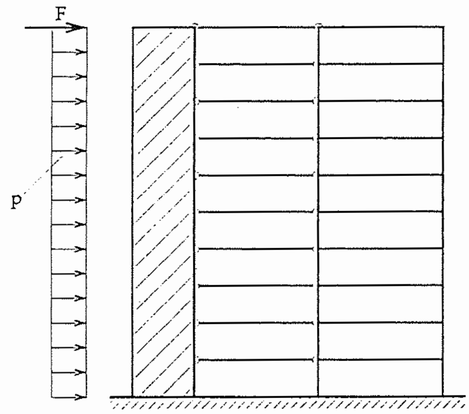

Figure 1 – Shear wall-frame plane association

At the base, under horizontal loading, the frame by itself has the major part of its lateral deformation and, as the wall, in that region, is much less deformed, it turns out that it is advantageous to have them associated by pinned-bars. On the other hand, at the top the wall is more deformed and the frame is not, and once more it is very convenient to have them associated to form tall building structures. The association produces the so called structural panel

That structural system has been successfully used during decades throughout the world, so allowing high rise buildings to be constructed, due to its high performance in resisting lateral forces caused by wind actions.

With respect to the structural analysis of such panels, Stamato (1966), in a pioneer work, carefully studied it, however, as was common practice at that occasion, neglecting axial deformations of the frame columns (that is, the frame deformation due to bending moment). The wall deformation due to shear force was also neglected, for being of small value. The procedure used the continuous medium technique by supposing panels to be horizontally linked by pinned-bars, along all the height of the building.

Battistelle (1991), in her Master Thesis, presented to the Department of Structural Engineering, São Carlos School of Engineering – USP, by using the continuous approximation, has considered, in the structural analysis of such panels, those columns axial deformations, as was also done in a previous work by Battistelle e Mancini (1989).

Those works, where uniform structural stiffness was supposed, have shown a big difference (through an example with twenty floors, equally spaced) in displacements of the building when considering the frame deformation by bending moment. Differences observed in the bending moments and shear forces in the frame and in the wall were insignificants. However, it was noticed that mathematic expressions were complex and the resulting calculations were very laborious, demanding the use of computers.

The procedure here proposed, of great simplicity, uses the continuous medium technique, and deals with structures of uniform geometry along the height of the building, with no difficulties at all, obtaining results in a straight manner, without loss of accuracy. So, another very useful tool for the tall building design engineer becomes available.

2. PROPOSED PROCEDURE

According to Battistelle (1991) and Battistelle e Mancini (1989), one knows that based on the continuous approximation, for one frame or for one wall, the horizontal displacements in the plane panel, due to lateral forces, may be evaluated at level z by using the expression

$$u_p = \int_0^z \frac{V_p}{s_p} dz + \int_0^z \int_0^z \frac{M_p}{j_p} dz dz \tag{1}$$

where $p \equiv w$ (wall) for a wall panel and $p \equiv f$ (frame) for a frame panel.

In Eq. (1) s_p is the shear force panel stiffness and j_p is its bending moment panel stiffness; V_p is the shear force and M_p is the bending moment, at level z, due to lateral loading, Fig. 2.

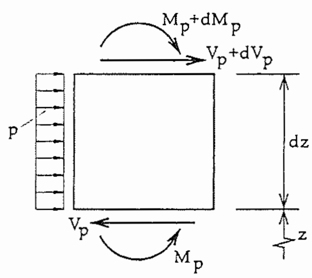

Figure 2 – Panel internal actions

When a frame is considered, $s_p = s_f$ and $j_p = j_f$, and for the case of two equal columns, according to Battistelle (1991), Battistelle and Mancini (1989), Mancini and Savassi (2001), are given by:

$$s_f = 24 \frac{E k_p k_v}{h(2k_p + k_v)} \tag{2}$$

and

$$j_f = \frac{E}{\left(\frac{1}{A_1} + \frac{1}{A_2}\right)} \ell_v^{\,2} \tag{3}$$

where

h = story height
I_p = column cross section second moment of area
k_p = column stiffness I_p/h
ℓ_v = beam span
I_v = beam cross section second moment of area
k_v = beam stiffness I_v/ℓ_v
A_1 e A_2 = frame column transverse areas
E = material longitudinal elasticity modulus

For the wall panel, of rectangular cross section of width equal to b and height equal to h, one has

$$s_w = \frac{GA}{c} = \frac{Gbh}{c} \tag{4}$$

and

$$j_w = \frac{Ebh^3}{12} \tag{5}$$

where G is the transverse elasticity modulus and c is the cross section shape coefficient.

In the case of the serial association, by pinned-bars, of two panels deformable by bending moment and shear force, one may write, for each of them, after differentiating twice Eq. (1):

$$s_1 u''_1 = -p_1 + \frac{s_1}{j_1} M_1 \tag{6}$$

and

$$s_2 u''_2 = -p_2 + \frac{s_2}{j_2} M_2 \tag{7}$$

where p_1 and p_2 are distributed horizontal forces and M_1 and M_2 are bending moments, at level z, of panels 1 and 2, respectively.

By adding Eq. (6) and Eq. (7) and taking into account that $u_1 \equiv u_2 \equiv u$, as a condition imposed by the continuous connection, one has

$$(s_1 + s_2)u'' = -p + \frac{s_1}{j_1} M_1 + \frac{s_2}{j_2} M_2 \tag{8}$$

where

$$p = p_1 + p_2 \tag{9}$$

is the distributed lateral force, due to horizontal lateral loading at level z.

From Eqs. (6), (7) and (9) it results

$$\frac{s_1}{s_1 + s_2}\left[-p + \frac{s_1}{j_1}(M - M_2) + \frac{s_2}{j_2} M_2\right] = -(p - p_2) + \frac{s_1}{j_1}(M - M_2) \tag{10}$$

where M is the bending moment due to all the lateral external forces evaluated at level z, and

$$M = M_1 + M_2 \tag{11}$$

From the rotational equilibrium of the infinitesimal element, shown in Fig. 2, one has:

$$p_2 = \frac{d^2 M_2}{dz^2} \tag{12}$$

From Eq. (10) and taking into account Eq. (12) the second order differential equation in M_2 is found

$$-M_2'' + \alpha^2 M_2 = -\beta_1 p + \beta_2 M \tag{13}$$

where

$$\alpha^2 = \frac{s_1 s_2}{(s_1 + s_2)}\left(\frac{1}{j_1} + \frac{1}{j_2}\right) \tag{14-a}$$

$$\beta_1 = \frac{s_2}{(s_1 + s_2)} \tag{14-b}$$

$$\beta_2 = \frac{s_1 s_2}{(s_1 + s_2)}\left(\frac{1}{j_1}\right) \tag{14-c}$$

According with Battistelle and Mancini (1997), for a lateral loading due to a uniformly distributed load p and a concentrated load F at the top, one finds

$$M_2 = C_1 e^{\alpha z} + C_2 e^{-\alpha z} - \frac{1}{\alpha^2}\left(Az^2 + Bz + C + \frac{2A}{\alpha^2}\right) \tag{15}$$

where

$$A = -\frac{p}{2}\beta_2 \tag{16-a}$$

$$B = (pH + F)\beta_2 \tag{16-b}$$

$$C = \beta_1 p - \beta_2\left(FH + p\frac{H^2}{2}\right) \tag{16-c}$$

Boundary conditions for Eq. (13) are, naming H the structure height:

$$M_2(H) = 0 \tag{17-a}$$

$$u_1(z) = u_2(z) \quad \text{for every } 0 \le z \le H \tag{17-b}$$

The condition (17-b) may be replaced by

$$M_2'(0) = -V_2(0) = -\frac{s_2}{s_1 + s_2} V(0) \tag{18}$$

that comes out from differentiation of Eq. (1), written for each panel, and evaluated at the base ($z=0$), that is:

$$u_1'(0) = u'(0) = \frac{V_1(0)}{s_1} \tag{19-a}$$

and

$$u_2'(0) = u'(0) = \frac{V_2(0)}{s_2} \tag{19-b}$$

By using

$$V = V_1 + V_2 \tag{20}$$

where V is the shear force due to the total lateral loading at level z, Eq. (20), together with Eqs. (19-a) and (19-b), leads to Eq. (18).

To calculate the integration constants C_1 and C_2 conditions (17-a) and (18) are used and then one may find the internal actions in the continuous structure:

$$M_1 = \frac{p}{2}z^2 - (pH + F)z + FH + p\frac{H^2}{2} - M_2 \tag{21}$$

$$V_2 = -\alpha C_1 e^{\alpha z} + \alpha C_2 e^{-\alpha z} + \frac{1}{\alpha^2}(2Az + B) \tag{22}$$

$$V_1 = p(H - z) + F - V_2 \tag{23}$$

$$p_2 = \alpha^2 C_1 e^{\alpha z} + \alpha^2 C_2 e^{-\alpha z} - \frac{2A}{\alpha^2} \tag{24}$$

$$p_1 = p - p_2 \tag{25}$$

If, in Eq. (7), one makes $s_2 = s_w \to \infty$, $j_2 = j_w$

it results

$$u'' = \frac{M_2}{j_2} = \frac{M_w}{j_w} \tag{26}$$

and then

$$M_2 = j_w u'' \tag{27}$$

If, in addition, one makes $s_1 = s_f$ and $j_1 = j_f$, then Eq. (13) in M_2 (wall bending moment) will become

$$\frac{j_w}{s_f}u^{iv} - \left(1+\frac{j_w}{j_f}\right)u'' = -\frac{V'}{s_f} - \frac{M}{j_f} \tag{28}$$

which is the differential equation already derived in Battistelle (1991), Battistelle and Mancini (1989), Mancini and Savassi (2001), for the shear wall-frame series association, considering the wall as extremely rigid to shear force action.

In terms of M_2 Eq. (28) may be written as

$$\frac{1}{s_f}M_2'' - \left[\frac{1}{j_f}+\frac{1}{j_w}\right]M_2 = \frac{-V'}{s_f} - \frac{M}{j_f} \tag{29}$$

Instead of Eq. (13), by following another way, one may get:

$$-\frac{M_2''}{s_2} + \frac{M_2}{j_2} = \frac{1}{(s_1+s_2)}\left[-p+\lambda_1 M+(\lambda_2-\lambda_1)M_2\right] \tag{30}$$

where

$$\lambda_1 = \frac{s_f}{j_f}; \quad \lambda_2 = \frac{s_w}{j_w} \tag{31}$$

3. EXAMPLE

To illustrate the procedure consider the structure of Fig. 1, composed by the association by pinned-bars of one wall and one frame. The rectangular wall cross section dimensions are *0.2m x 1.5m*. The frame has columns and beams of rectangular cross section with dimensions, constant through the building height, of *0.4m x 0.4m* and *0.2m x 0.4m*, respectively. Beam length is $\ell_v = 4.0m$. Story height is constant and equals *3.0m*. Total building height is *60.0m*. The material is reinforced concrete, with Young's modulus $E = 2.0 \times 10^7\ kN/m^2$ and Poisson coefficient equal to *0.16*.

Loading is due to a lateral uniformly distributed force $p = 0.4kN/dm$, along the height of the building and the concentrated Force F is zero.

Determination of the stiffness parameters:

a) wall

$$I_w = 2x\frac{15^3}{12} = 562{,}5dm^4$$

$$j_2 = j_w = EI_w = 2.0 \times 10^5 \times 562.5 = 11.25 \times 10^8\ kNdm^2$$

$$G = \frac{E}{2(1+\nu)} = \frac{2.0 \, x \, 10^5}{2(1+0.16)} = 0.862 \, x \, 10^5 \, kN \, / \, dm^2$$

$A = 2 \; x \; 15 = 30 \; dm^2$

c= *1.2* (cross section shape coefficient)

$$s_2 = s_w = \frac{GA}{c} = 21.55 \, x \, 10^5 kN$$

b) frame

$$I_p = \frac{4 \, x \, 4^3}{12} = 21.333 dm^4$$

$$k_p = \frac{I_p}{h} = \frac{21.333}{30} = 0.7111 dm^3$$

$$I_v = \frac{2 \, x \, 4^3}{12} = 10.666 dm^4$$

$$k_v = I_v / \ell_v = \frac{10.666}{40} = 0.2667 dm^3$$

$$s_f = 24 \frac{E}{h} \frac{k_p k_v}{(2k_p + k_v)} = \frac{24 \, x \, 2.0 \, x \, 10^5 \; x \; 0.7111 \, x \; 0.2667}{30 \, x (2 \, x \; 0.7111 + 0.2667)} = 17966.8 kN$$

$$j_f = E \ell_v^2 \frac{1}{\left(\frac{1}{A_1} + \frac{1}{A_2} \right)} = \frac{2.0 \, x \, 10^5 \; x \; 40^2}{\left(\frac{1}{16} + \frac{1}{16} \right)} = 2.56 \, x \, 10^9 \, kN.dm^2$$

Then, it results:

$$\lambda_1 = \frac{s_f}{j_f} = 7.01836 \, x \, 10^{-6}$$

$$\lambda_2 = \frac{s_w}{j_w} = 1.91555 \, x \, 10^{-2}$$

By substituting in Eq. (30), it results:

$$-\frac{1}{21.55 \, x \, 10^5} \left(\frac{d^2 M_2}{dz^2} \right) + \frac{M_2}{1.125 \, x \, 10^8} = \frac{1}{(0.17967 \, x \, 10^5 + 21.55 \, x \, 10^5)} \; .$$

$$[-p + 7.01836 \times 10^{-6} M + (1.91555 \times 10^{-2} - 7.01836 \times 10^{-6})M_2]$$

or

$$-4.64037 \times 10^{-7} M''_2 + 7.675 \times 10^{-11} M_2 = -4.6020 \times 10^{-7} p + 3.22985 \times 10^{-12} M$$

or

$$-M''_2 + 1.65396 \times 10^{-4} M_2 = -9.91731 \times 10^{-1} p + 6.96033 \times 10^{-6} M$$

Then, according to the proposed procedure:

$$\alpha^2 = 1.65396 \times 10^{-4} \quad \rightarrow \qquad \alpha = 1.29606 \times 10^{-2}$$

$$\beta_1 = 0.99173 \qquad \beta_2 = 6.96033 \times 10^{-6}$$

With loading values

$p = 0.4kN/dm$ and $F = 0$

then, it follows

$$A = -1.39207 \times 10^{-6}$$
$$B = 1.67048 \times 10^{-3}$$
$$C = -1.04452 \times 10^{-1}$$

By substitution in Eq. (15) it results

$$M_2 = C_1 e^{1.28606 x 10^{-2} z} + C_2 e^{-1.28606 x 10^{-2} z} + 8.35613 x 10^{-3} z^2 - 10.0999 z + 6.31527.10^2 + 101.775$$

$$V_2 = -M'_2 = -1.28606 x 10^{-2} C_1 e^{1.28606 x 10^{-2} z} + 1.28606 x 10^{-2} C_2 e^{-1.28606 x 10^{-2} z} - 1.67123 x 10^{-2} z + 10.0999$$

Imposing, now, the boundary conditions:

a) $M_2(600) = 0$

b) $V_2(0) = \dfrac{21.55 x 10^5}{0.17967 x 10^5 + 21.55 x 10^5} V(0) = 0.99173 V(0) =$

$$0.99173 \times 0.4 \times 600 = 238.016$$

it follows

$$C_1 = 1.02930 \quad ; \qquad C_2 = 17723.64$$

Figure 3 and Fig. 4 graphically show the bending moments and shear forces in the wall and in the frame.

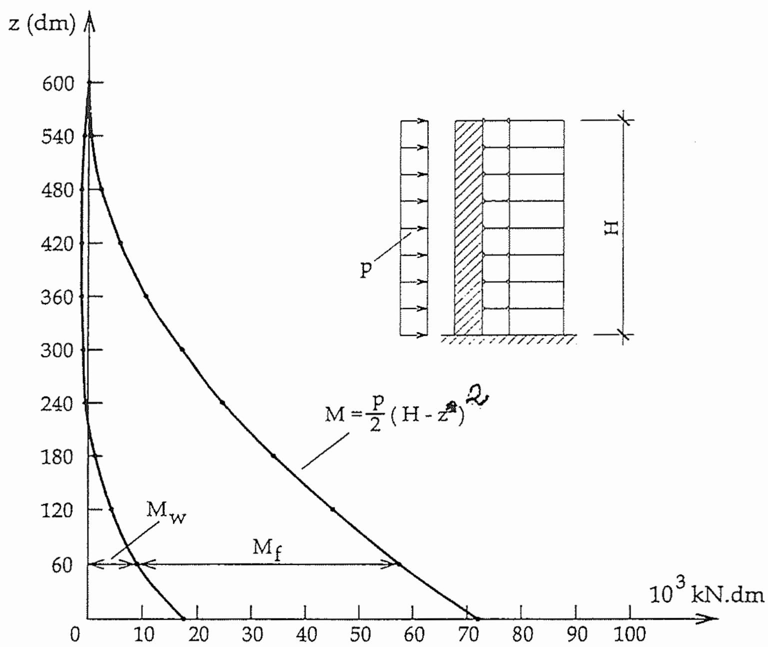

Figure 3 – Wall and frame bending moments

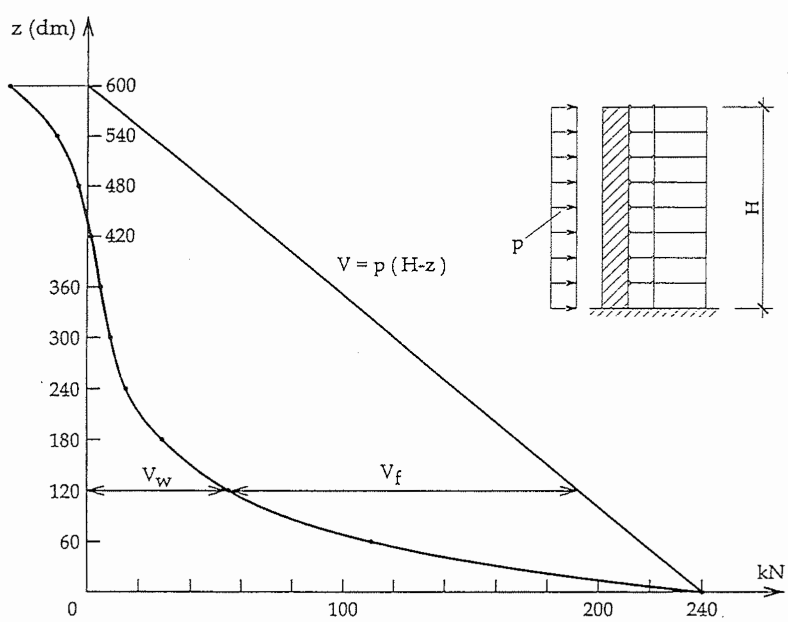

Figure 4 – Wall and frame shear forces

Figure 5 represents the panel horizontal displacements, Eq. (1), considering and neglecting columns axial deformations and Table (1) presents the results obtained for horizontal displacements according with three procedures: Battistelle (1991), discrete analysis and the present proposed procedure.

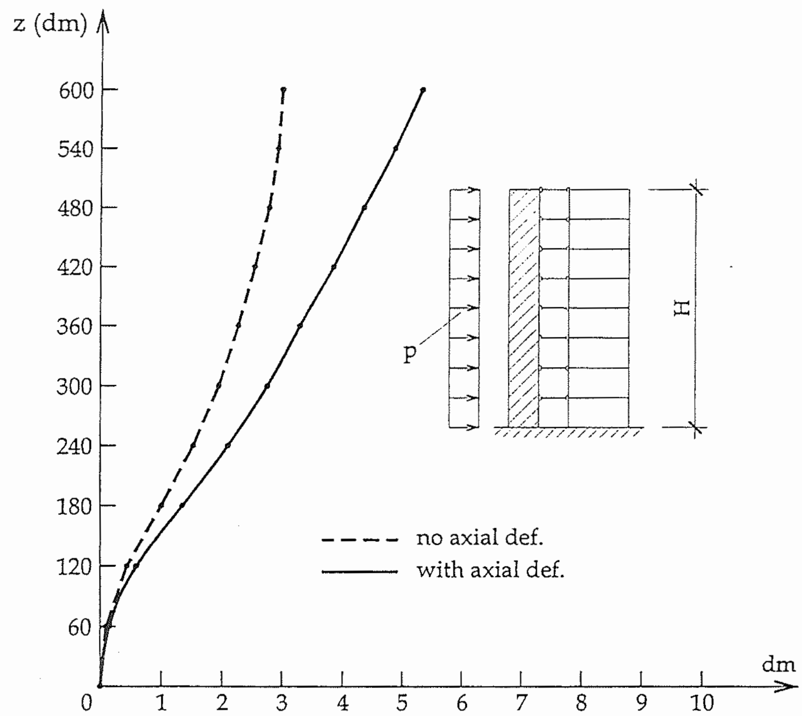

Figure 5 – Panel horizontal displacements

Tabel 1 – Horizontal displacements

z (dm)	Battistelle (1991) (m)	Discrete (m)	Present procedure (m)
0	0.0000	0.0000	0.0000
60	0.0232	0.0229	0.0235
120	0.0745	0.0740	0.0749
180	0.1375	0.1371	0.1378
240	0.2037	0.2039	0.2042
300	0.2688	0.2699	0.2693
360	0.3305	0.3326	0.3309
420	0.3873	0.3905	0.3876
480	0.4386	0.4429	0.4389
540	0.4848	0.4899	0.4849
600	0.5273	0.5320	0.5272

As seen in Tab. 1, results are entirely satisfactory and, also, values obtained for the internal actions in the panels have a good coincidence to those of Battistelle (1991).

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AVALIAÇÃO DOS DESLOCAMENTOS LATERAIS DE PAINÉIS PLANOS GERAIS DE EDIFÍCIOS ALTOS SUJEITOS ÀS FORÇAS HORIZONTAIS UTILIZANDO A TÉCNICA DO MEIO CONTÍNUO

RESUMO

Neste trabalho calculamos os deslocamentos laterais de um painel geral, típico de estruturas de edifícios altos, quando sujeito às forças horizontais.

Neste cálculo, aproximamos o painel geral pela associação plana de um pórtico e um pilar-parede por barras bi-articuladas.

Os resultados obtidos, com um esforço pequeno de cálculo, são bastante satisfatórios quando comparados com os valores fornecidos por um programa de computador, desenvolvido utilizando a aproximação discreta de análise.

I – INTRODUÇÃO

Em trabalho anterior de MANCINI e SAVASSI[1] foi mostrado, utilizando a técnica do meio contínuo, que a associação de duas paredes por vigas e a associação de uma parede e um pórtico por barras bi-articuladas são estruturas equivalentes do ponto de vista do comportamento estrutural, quando as mesmas estão sujeitas a forças horizontais.

Observamos que esta equivalência foi mostrada levando em conta nas deduções as deformações axiais dos pilares.

Desta forma, a referida associação de pilares-parede pode ter seus deslocamentos horizontais, quando sujeita a um carregamento horizontal, avaliados através do cálculo destes deslocamentos na associação de parede e pórtico, equivalente à primeira destas estruturas.

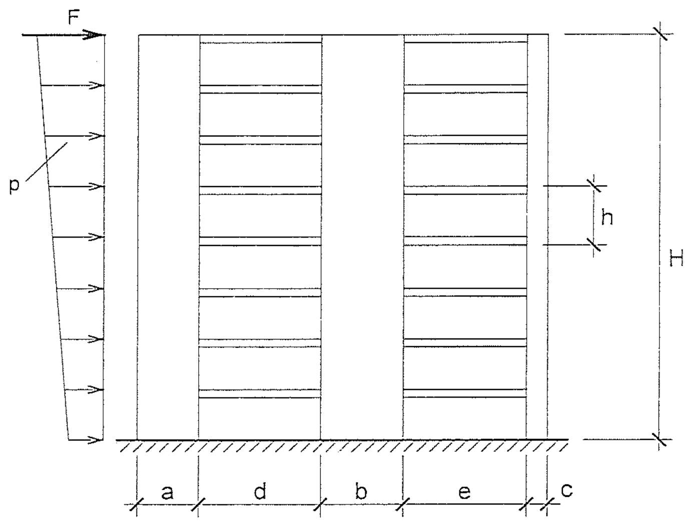


Figura 1 – Painel Geral

No caso de um painel geral, como a da figura (1), que será estudado no presente trabalho, o referido painel tem, também, o seu comportamento estático, sob forças horizontais, aproximadamente equivalente ao da associação de uma parede e um pórtico por barras bi-articuladas. Notamos que esta equivalência subsiste, de forma exata, quando se desprezam as deformações axiais dos pilares por força normal.

Segundo MANCINI[2], com esta última hipótese simplificadora, para qualquer painel de edifício alto, constituído de pilares de pórtico (onde só consideramos a flexão local entre os andares), pilares-parede (onde só consideramos a flexão global do conjunto), e vigas horizontais, vale a expressão

$$V = -j_g u''' + s_g u'$$

onde

V é a força cortante do carregamento lateral, u denota o deslocamento horizontal à cota z, j_g é a soma dos produtos de rigidez à flexão dos pilares-parede e s_g é a rigidez de painel à força cortante.

A equação anterior é válida também para a associação de pórtico e pilar-parede por barras bi-articuladas, desprezando as deformações axiais dos pilares do pórtico. A avaliação dos deslocamentos horizontais pode então ser realizada calculando a estrutura equivalente, o que demonstramos ser bastante simples, utilizando a técnica do meio contínuo.

No caso de uma estrutura tridimensional de edifício alto, quando se deseja estudar a distribuição das forças horizontais entre os painéis de contraventamento, o referido painel geral pode ser substituído pelo painel equivalente pórtico-parede, de mesma rigidez, permitindo simplificar consideravelmente os cálculos, conforme MANCINI[2], MANCINI E SAVASSI[3].

Como no caso do painel constituído por dois pilares-parede por vigas, considerando às deformações axiais dos pilares, haverá três parâmetros que definem o comportamento do painel geral:

1) j_g é a soma dos produtos de rigidez à flexão EI de todos os pilares-parede

2) s_g é a rigidez do painel à força cortante

3) j_f é o produto EI_f onde I_f é o momento de inércia total das seções transversais de todos os pilares em relação ao centróide do conjunto, desprezando entretanto os momentos de inércia destas seções em relação aos seus próprios centróides.

   E é o módulo de Young do material

A estrutura equivalente (associação pórtico-parede por barras bi-articuladas), terá os parâmetros

a) $j_w = j_g$ (produto de rigidez ã flexão da parede)

b) $s_f = s_g$ (rigidez do pórtico à força cortante)

c) j_f (parâmetro já definido anteriormente correspondente aos pilares do pórtico)

II. DEDUÇÃO DO PARÂMETRO S_g

Os parâmetro j_g e j_f são por demais evidentes. O parâmetro s_g do painel geral é a rigidez à força cortante do mesmo considerando os seus pilares indeformáveis axialmente. Este parâmetro está apresentado na tabela (1), que adiantamos agora, sendo que as deduções serão feitas a seguir. Esta tabela refere-se à estrutura da figura (2) relativa a um painel qualquer.

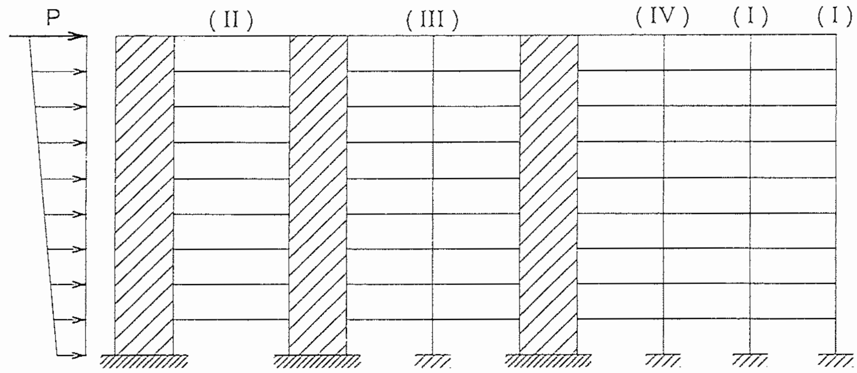


Figura 2 – Painel qualquer

Observamos na figura (2), que o número romano (I) indica prumadas de pilar de pórtico não ligados diretamente às paredes.

A indicação em (II) romano refere-se à prumada de um conjunto de vigas ligadas diretamente às paredes vizinhas, não havendo aí, neste trecho, nenhum pilar de pórtico. A prumada tipo (III) indica uma prumada de pilar de pórtico entre duas paredes e, finalmente, a prumada tipo (IV) constitui-se em um pilar de pórtico ligado por vigas a uma parede.

Cada tipo de prumada anterior contribuirá para o valor da rigidez à força cortante s_g de modo que

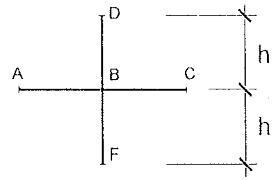
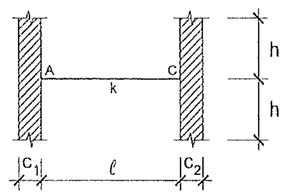
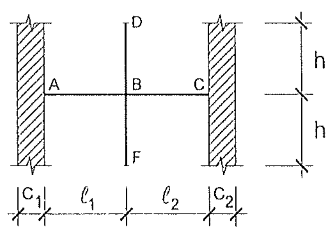
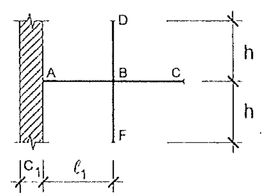
$$s_g = s'_g + r_g \qquad \text{(II.1)}$$

conforme veremos a seguir.

A dedução deste parâmetro foi feita pela primeira vez por CARDAN[4] com algumas incorreções apontadas por MANCINI[2].

Aqui, para a deduções seguiremos o excelente trabalho de FERREIRA[5], de onde extraímos os itens seguintes:

TABELA 1 – Contribuições para s'_g e r_g

TIPO	ESQUEMA	s'_g	r_g	OBSERVAÇÕES
I		$\frac{12E}{h}k_F\frac{k_A+k_C}{\sum k}$ Onde $\sum k = k_A + k_C + k_D + k_F$	Não contribui	Quando não existir o tramo AB (ou BC) será válida a mesma expressão fazendo-se $k_A = 0$ (ou $k_c = 0$)
II		Não contribui	$6E\bar{k}\left(1+\frac{c_1+c_2}{2\ell}\right)\left(1+\frac{c_1}{\ell}\right)$ para parede da esquerda onde $\bar{k} = \frac{k}{h}$	Para a parede da direita será válida a mesma expressão trocando-se o índice "1" por "2", "A" por "C", e vice-versa.
III		$\frac{18E}{h}\left[\left(1+\frac{c_1}{2\ell_1}\right)k_A+\left(1+\frac{c_2}{2\ell_2}\right)k_C\right]\frac{k_F}{\sum k}$ onde $\sum k = k_A + k_C + 1{,}5k_D + 1{,}5k_F$	$6E\bar{k}_A\left\{\left(1+\frac{c_1}{2\ell_1}\right)\left(1-\frac{k_A}{\sum k}\right)-\left(1+\frac{c_2}{2\ell_2}\right)\frac{k_C}{2\sum k}+\right.$ $\left.+\frac{c_1}{2\ell_1}\left[\left(1+\frac{c_1}{2\ell_1}\right)\left(2-\frac{3k_A}{2\sum k}\right)-\left(1+\frac{c_2}{2\ell_2}\right)\frac{3k_C}{2\sum k}\right]\right\}$ Onde $\sum k = k_A + k_C + 1{,}5k_D + 1{,}5k_F$ e $\bar{k}_A = \frac{k_A}{h}$	Válidos para a parede da direita trocando-se o índice Ï" por "2", "A " por "C"" , e vice-versa
IV		$\frac{18E}{h}\left[\left(1+\frac{c_1}{2\ell_1}\right)k_A+k_C\right]\frac{k_F}{\sum k}$ Onde $\sum k = k_A + 1{,}5k_C + 1{,}5k_D + 1{,}5k_F$	$6E\bar{k}_A\left[\left(1+\frac{c_1}{2\ell_1}\right)\left(1+\frac{c_1}{\ell_1}\right)+\right.$ $\left.-\left(1+\frac{3c_1}{2\ell_1}\right)\frac{\left(1+\frac{c_1}{2\ell_1}\right)k_A+k_C}{2\sum k}\right]$ onde $\sum k = k_A + 1{,}5k_C + 1{,}5k_D + 1{,}5k_F$	Quando não existir o tramo BC, serão válidas as mesmas expressões fazendo-se $k_C = 0$

II-1 – Prumadas do tipo (I)

Para a dedução do parâmetro s'_g do pilar de pórtico não ligado a parede consideremos a figura (3) onde as rotações de três nós consecutivos, tanto na viga como no pilar, são consideradas iguais

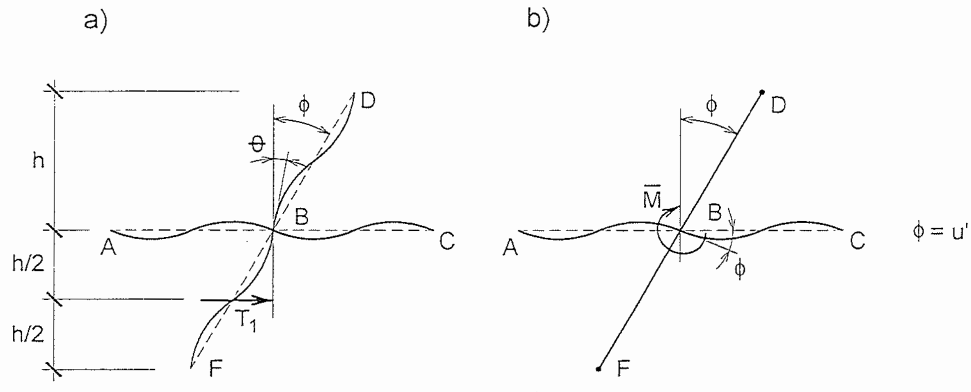


Figura 3 – Contribuição para s_g do pilar de pórtico

Convencionamos como positivos os momentos anti-horários que as barras aplicam aos nós e designamos com a letra <u>k</u> as "rigidezes" $\frac{I}{\ell}$ das barras onde ℓ é o vão das mesmas e I o seu momento de inércia.

Para que os pilares fiquem livres de esforços aplicamos ao nó o momento externo $\overline{M}$.

Este momento impõe a rotação ϕ sem curvatura do pilar (figura (3-b)). Liberando o nó B, o que significa aplicar um momento igual e de sentido contrário a $\overline{M}$, as barras que aí concorrem recebem parcelas de momento proporcionais às rigidezes de cada uma.

De acordo com a configuração final da figura (3-b) temos

$$\overline{M} = 6E(k_A + k_C)\phi \qquad \text{(II.2)}$$

e então

$$M_{BF} = \overline{M}\frac{k_F}{\sum k} \qquad \text{(II-3)}$$

onde

$$\sum k = k_A + k_C + k_D + k_F \qquad \text{(II.4)}$$

Sendo a força cortante no pilar dada por

$$T_1 = 2\frac{M_{BF}}{h} \qquad \text{(II.5)}$$

concluímos, pelas equações (II-3) e (II-5) que

$$T_1 = s_g^{'}\,\phi = s_g^{'}\,u' \qquad \text{(II-6)}$$

onde u denota o deslocamento horizontal u(z) e

$$s'_g = \frac{12E}{h}.k_F.\frac{k_A + k_C}{\sum k} \qquad \text{(II-7)}$$

representa a contribuição da prumada tipo (I) para a rigidez global do painel geral.

No caso, como supomos o pilar não ligado à paredes por vigas então $r_g = 0$, conforme ficará mais claro nas deduções a seguir

II-2 – Prumadas do tipo (II)

Tomemos a estrutura da figura (4) constituída por duas paredes unidas por vigas. Este painel caracteriza-se pela ausência das prumadas de pilares de pórtico e portanto a rigidez s'_g é nula.

Designamos novamente por k a rigidez $\frac{I}{\ell}$ das vigas. Estas são consideradas perfeitamente engastadas nas paredes e suas reações elásticas (m_1 e m_2) são consideradas distribuídas continuamente ao longo das mesmas, conforme a técnica contínua.

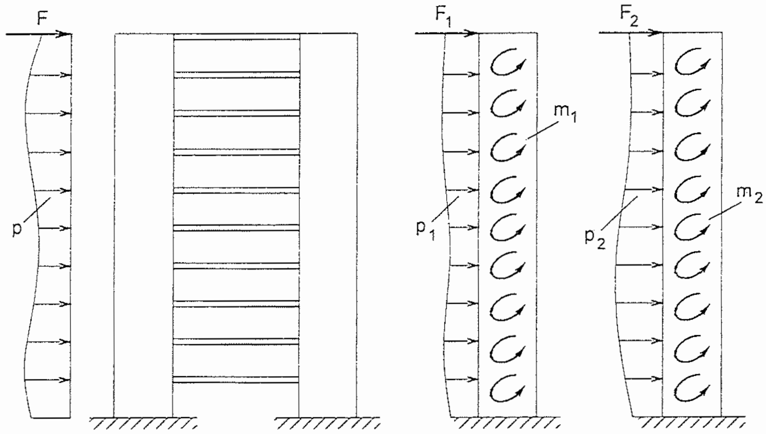


Figura 4 – Paredes unidas por vigas

O equilíbrio à rotação de um elemento infinitesimal da parede esquerda, com os esforços positivos obedecendo os sentidos indicados na figura (5), fornece

$$\frac{dM_1}{dz} = -V_1 + m_1 \tag{II-8}$$

Para a parede da direita, expressão análoga pode ser escrita:

$$\frac{dM_2}{dz} = -V_2 + m_2 \tag{II.9}$$

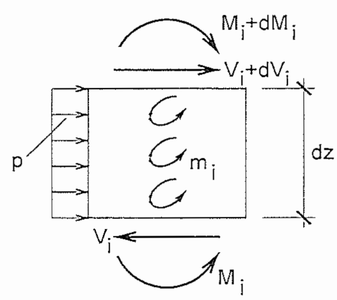


Figura 5 – Elemento genérico de parede

Desprezando as deformações por cisalhamento, a situação deformada do elemento é a esquematizada na figura (6) e os momentos fletores nas paredes são, respectivamente:

$M_1 = j_1\, u''$ (II-10a)

$M_2 = j_2\, u''$ (II-10b)

onde com j indicamos o produto de flexão EI e u denota o deslocamento horizontal à cota z.

Na deformação de um trecho do painel mostrada na figura (6), observamos que não são aí incluídas as deformações dos pilares por força normal.

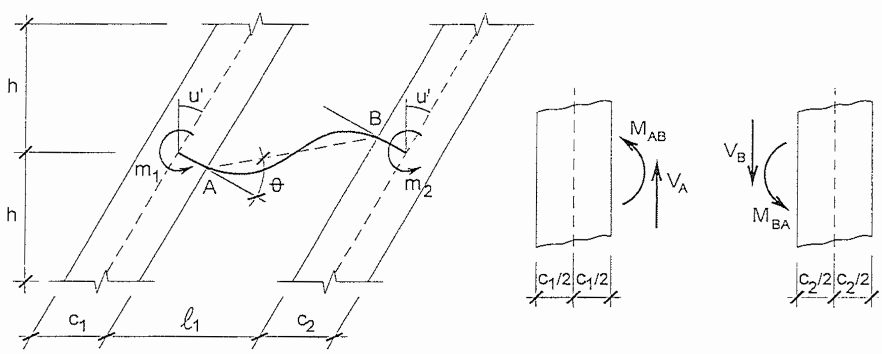


Figura 6 – Deformação do painel

Vemos pela figura (6) que

$$\theta = \left(1 + \frac{c_1 + c_2}{2\ell_1}\right)u' \qquad \text{(II-11)}$$

e temos também

$$M_{AB} = M_{BA} = 6Ek_1\left(1 + \frac{c_1 + c_2}{2\ell_1}\right)u' \qquad \text{(II-12)}$$

onde k_1 denota a rigidez $\frac{I}{\ell}$ das vigas.

O equilíbrio da barra fornece

$$V_A = V_B = \frac{M_{AB} + M_{BA}}{\ell_1} \qquad \text{(II.13)}$$

e portanto

$$V_A = V_B = \frac{12Ek_1}{\ell_1}\left(1 + \frac{c_1 + c_2}{2\ell_1}\right)u' \qquad \text{(II.14)}$$

Calculamos agora os momentos distribuídos m_1 e m_2 através da continuização dos esforços concentrados ao nível dos andares

$$hm_1 = M_{AB} + V_A \frac{c_1}{2} \tag{II.15a}$$

$$hm_2 = M_{BA} + V_B \frac{c_2}{2} \tag{II.15b}$$

As expressões (II.15a) e (II.15b) junto com (II.12) e (II.14) fornecem

$$m_1 = r_1\, u' \tag{II.16a}$$

$$m_2 = r_2\, u' \tag{II.16b}$$

onde

$$r_1 = 6E\bar{k}_1\left(1+\frac{c_1+c_2}{2\ell_1}\right)\left(1+\frac{c_1}{\ell_1}\right) \tag{II.17a}$$

e

$$r_2 = 6E\bar{k}_1\left(1+\frac{c_1+c_2}{2\ell_1}\right)\left(1+\frac{c_2}{\ell_2}\right) \tag{II.17b}$$

onde $\bar{k}_1$ denota a rigidez $k_1 = \frac{I}{\ell_1}$ das vigas, por unidade de altura, ou seja, a rigidez do meio contínuo à flexão, dada por

$$\bar{k}_1 = \frac{k_1}{h} \tag{II.18}$$

Introduzindo as equações (II.16a) e (II.16b) em (II.8) e (II.9) obtemos

$$V_1 = -\, j_1\, u''' + r_1\, u' \tag{II.19a}$$

$$V_2 = -\, j_2\, u''' + r_2\, u' \tag{II.19b}$$

onde V_1 e V_2 são as forças cortantes nas paredes.

Pelo equilíbrio global da estrutura podemos escrever

$$V = V_1 + V_2 = -\,(j_1 + j_2)u''' + (r_1 + r_2)u' \tag{II.20}$$

ou

$$V = -\, j_g\, u''' + r_g\, u' \tag{II.21}$$

que é também a equação da associação pórtico-parede por barras bi-articuladas desprezando as deformações axiais dos pilares do pórtico.

Na equação anterior

$$r_g = r_1 + r_2 \tag{II.22}$$

e V é a força cortante total do carregamento lateral à cota z.

Efetuando a soma indicada na equação (II.22) encontramos

$$s_g = \frac{3}{2}\frac{EI_v}{h}\frac{(\bar{c}_1 + +\bar{c}_2)^2}{\left(\frac{\ell_1}{2}\right)^3} \tag{II.23}$$

valor encontrado também em MANCINI[2], onde I_v é o momento de inércia das vigas e

$$\bar{c}_1 = \frac{c_1 + \ell_1}{2} \tag{II.24a}$$

$$\bar{c}_2 = \frac{c_2 + \ell_2}{2} \tag{II.24b}$$

II.3 – Prumadas do tipo (III)

Consideremos agora o tipo de associação mostrado na figura (7) em que temos três tipos de elementos: os pilares paredes (1 e 2), uma prumada de pilar de pórtico (3) e vigas adajacentes às paredes. Estes elementos contribuirão para as rigidezes "j_g", "s_g" e "r_g", características do painel resultante.

As paredes, neste painel, comportam-se elasticamente de mesma maneira que no exemplo do item anterior. Desta maneira as equações (II.19a) e (II.19b) continuam válidas e serão reescritas determinando as constantes r_1 e r_2 para o presente caso de associação.

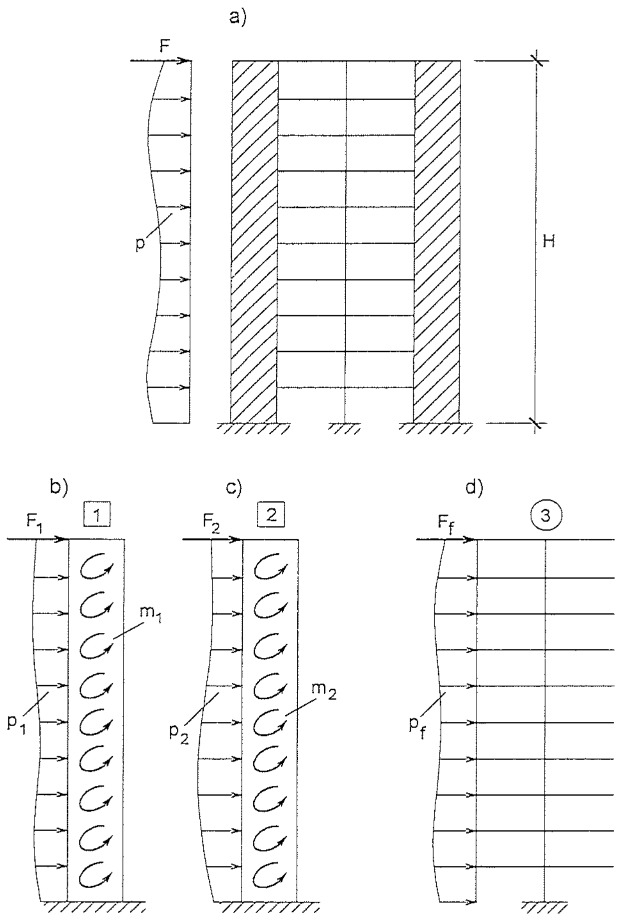


Figura 7 – Pilar de pórtico ligado a duas paredes

$$V_1 = - j_1 \, u''' + r_1 \, u' \qquad (II.25a)$$

$$V_2 = - j_2 \, u''' + r_2 \, u' \qquad (II.25b)$$

Para o pilar de pórtico (3) determinaremos a constante de rigidez s'_g que relaciona a força cortante em um determinado nível com a distorção u', conforme a relação abaixo.

$$V_f = s'_g \, u' \qquad (II.26)$$

relação cuja validade demonstraremos a seguir

A força cortante externa será equilibrada pelas forças cortantes atuantes em cada elemento vertical, isto é:

$$V = V_1 + V_z + V_f \qquad (II.27)$$

As equações (II.25a) e (II.25b), juntamente com a equação (II.26), substituídas em (II.27) fornecem

$$V = - j_g\, u''' + s_g\, u' \qquad (II.28)$$

onde

$$j_g = j_1 + j_2 \qquad (II.29a)$$

$$s_g = s'_g + r_g \qquad (II.29b)$$

onde r_g é obtido pela soma das contribuições r_1 e r_2 das vigas adjacentes às paredes.

A figura (8) mostra a deformação do andar genérico pelo efeito de carregamento externo

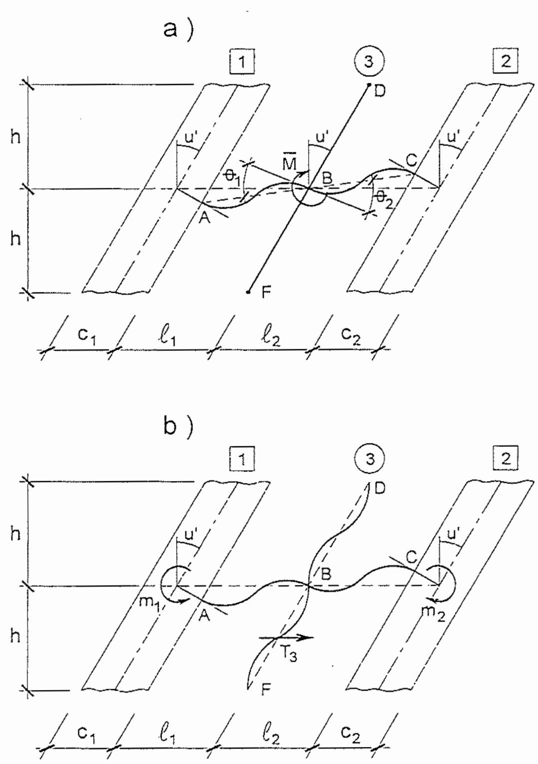


Figura 8 – Andar genérico do painel

Na figura (8a) o momento $\overline{M}$ é o momento de bloqueio aplicado ao nó B quando se impõe a rotação u', sem curvatura no pilar. A seguir o nó será liberado à rotação, segundo a técnica de CROSS, mantendo-se fixo o deslocamento do andar. Liberar o nó é equivalente e aplicar um momento igual de sentido contrário ao de $\overline{M}$, que será repartido entre as barras que concorrem neste nó, levando em conta a rigidez de cada uma.

A configuração final do trecho do painel (figura (8.b)) mostra que as vigas AB e BC permanecem engastadas nas paredes, definido as respectivas rigidezes: $4Ek_A$ e $4Ek_c$. Os nós D, B, F da prumada de pilar, na posição final, sofrerão rotações próximas que serão admitidas iguais. Assim, as rigidezes dos tramos BD e BF da prumada 3 são respectivamente $6Ek_D$ e $6Ek_F$.

O momento externo $\overline{M}$ valerá

$$\overline{M} = 6Ek_A\,\theta_1 + 6Ek_C\,\theta_2 \tag{II.30}$$

onde os ângulos θ_1 e θ_2 são indicados na figura (8.a) e são dados por

$$\theta_1 = \left(1+\frac{c_1}{2\ell_1}\right)u' \tag{II.31a}$$

e

$$\theta_2 = \left(1+\frac{c_2}{2\ell_2}\right)u' \tag{II.31b}$$

Logo o momento $\overline{M}$ será expresso por

$$\overline{M} = 6E\left[\left(1+\frac{c_1}{2\ell_1}\right)k_A + \left(1+\frac{c_2}{2\ell_2}\right)k_C\right]u' \tag{II.32}$$

Liberando o nó da ação do momento $\overline{M}$ obtemos

$$M_{BA} = 6Ek_A\left(1+\frac{c_1}{2\ell_1}\right)u' - \frac{k_A}{\sum k}\overline{M} \tag{II.33}$$

onde

$$\sum k = k_A + k_C + 1{,}5k_d + 1{,}5k_F \qquad (II.34)$$

Lembrando que o coeficiente da propagação de B para A é igual a 1/2, temos

$$M_{AB} = 6Ek_A\left(1+\frac{c_1}{2\ell_1}\right)u' - \frac{1}{2}\frac{k_A}{\sum k}\overline{M} \qquad (II.35)$$

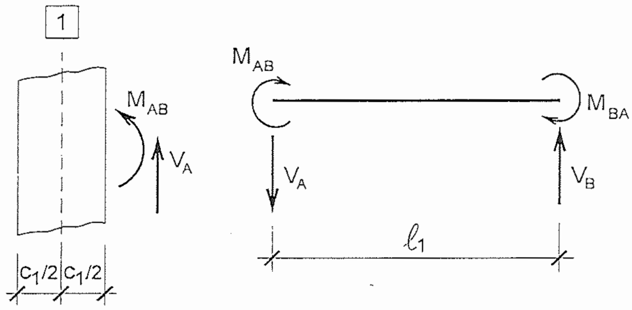


Figura 9 – Equilíbrio da viga AB

Conforme figura (9), o equilíbrio da barra (viga) AB implica em

$$V_A = \frac{M_{AB} + M_{BA}}{\ell_1} \qquad (II.36)$$

O efeito da flexão da viga AB sobre a parede 1, conforme a técnica contínua, é o momento m_1, distribuída segundo a altura do edifício, sendo dado por

$$hm_1 = M_{AB} + V_A\frac{c_1}{2} \qquad (II.37)$$

ou

$$m_1 = r_1 u' \qquad (II.38)$$

onde após as devidas substituições encontramos:

$$r_1 = 6E\overline{k}_A\left\{\left(1+\frac{c_1}{2\ell_1}\right)\left(1+\frac{k_A}{2\sum k}\right) - \left(1+\frac{c_2}{2\ell_2}\right)\frac{k_C}{2\sum k}' \right. +$$

$$\left. + \frac{c_1}{2\ell_1}'\left[\left(1+\frac{c_1}{2\ell_1}\right)\left(2-\frac{3k_A}{2\sum k}\right) - \left(1+\frac{c_2}{2\ell_2}\right)\frac{3k_C}{2\sum k}\right]\right\} \qquad (II.39)$$

Procedendo de maneira inteiramente análoga para a viga BC encontramos a constante r_2, sendo ainda válida a expressão (II.39) trocando os índices "A "por "C" e "1" por "2" nas grandezas correspondentes.

Para o cálculo de s'_g tomamos um elemento de pilar de pórtico 3, na posição deformada final, mostrada na figura (10)

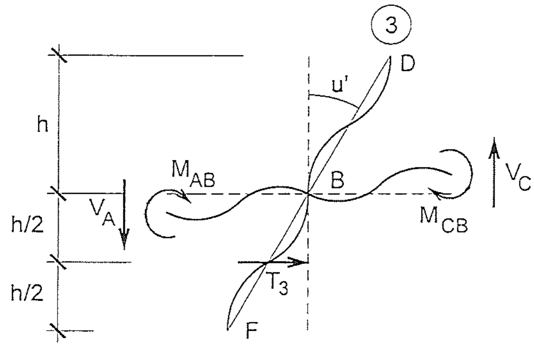


Figura 10 – Elemento do pilar

O momento M_{BF} no tramo abaixo do andar considerado vale

$$M_{BF} = 1{,}5 \frac{k_F}{\sum k} \overline{M} \tag{II.40}$$

onde, usando as equações (II.32) e (II.34), obtemos

$$M_{BF} = \frac{9Ek_F}{\sum k} \left[\left(1 + \frac{c_1}{2\ell_1} \right) k_A + \left(1 + \frac{c_2}{2\ell_2} \right) k_C \right] u' \tag{II.41}$$

A força cortante no tramo BF vale

$$T_3 = 2 \frac{M_{BF}}{h} \tag{II.42}$$

E tendo em vista a equação (II.41) encontramos

$$T_3 = s'_g \, u' \tag{II.43}$$

ou $$s'_g = \frac{18E}{h} \left[\left(1 + \frac{c_1}{2\ell_1} \right) k_A + \left(1 + \frac{c_2}{2\ell_2} \right) k_C \right] \frac{k_F}{\sum k} \tag{II.44}$$

II.4 – Prumadas do tipo (IV)

Para não alongarmos demais este trabalho mostraremos a deformação de um trecho do painel e deixaremos ao leitor a demonstração das fórmulas que estão registradas na Tabela (1).

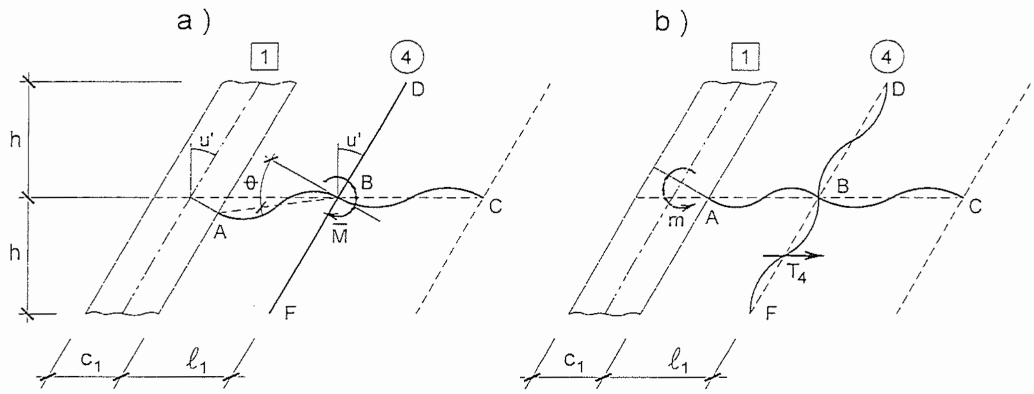


Figura 11 – Andar genérico

Com base na figura (11) são feitas as correspondentes deduções, encontrando

$$r = 6E\bar{k}_A\left[\left(1+\frac{c_1}{2\ell_1}\right)\left(1+\frac{c_1}{\ell_1}\right)-\left(1+\frac{3c_1}{2\ell_1}\right)\frac{\left(1+\frac{c_1}{2\ell_1}\right)k_A+k_C}{2\sum k}\right] \qquad (II.45)$$

e

$$s'_g = \frac{18E}{h}\left[\left(1+\frac{c_1}{2\ell_1}\right)k_A + k_C\right]\frac{k_F}{\sum k} \qquad (II.46)$$

Observamos que, no caso particular de não termos a viga BC, valem as mesmas fórmulas anteriores, tomando $k_C = 0$.

III. A ESTRUTURA EQUIVALENTE

Vimos em MANCINI e SAVASSI[1] que a equação diferencial da associação parede-pórtico, nos deslocamentos horizontais do conjunto se escreve

$$\frac{j_w}{s_f} u^{IV} - \left(1 + \frac{j_w}{j_f}\right) u'' = -\frac{V'}{s_f} - \frac{M}{j_f} \qquad \text{(III.1)}$$

onde

V e M são, respectivamente, a força cortante e o momento fletor do carregamento à cota z.

Se desprezarmos as deformações da parede por força cortante vale:

$$M_w = j_w \, u'' \qquad \text{(III.2)}$$

onde M_w é o momento fletor na parede à cota z, de onde, substituindo em (III.1), ficamos com

$$\frac{M_w''}{s_f} - \left(\frac{1}{j_w} + \frac{1}{j_f}\right) M_w = -\frac{V'}{s_f} - \frac{M}{j_f} \qquad \text{(III.3)}$$

Para a solução da equação (III.3) devemos colocá-la na forma

$$-M''_w + \alpha^2 M_w = -\beta_1 p + \beta_2 M \qquad \text{(III.4)}$$

onde p é o valor da força distribuída do carregamento a cota z e p = - V'.

Conforme BATTISTELLE e MANCINI[6] , para um carregamento constituído por uma força uniformemente distribuída p em toda a altura H do painel e uma força horizontal concentrada no topo, de valor F, encontramos

$$M_w = C_1 e^{\alpha z} + C_2 e^{-\alpha z} - \frac{1}{\alpha^2}\left(Az^2 + Bz + C + \frac{2A}{\alpha^2}\right) \qquad \text{(III.5)}$$

onde

$$A = -\frac{p}{2} \beta_2 \qquad \text{(III.6a)}$$

$$B = (pH + F)\beta_2 \tag{III.6b}$$

$$C = \beta_1 \, p - \beta_2 \left(FH + p\frac{H^2}{2} \right) \tag{III.6c}$$

As condições de contorno para a equação diferencial (III.4) são

$$M_w \, (H) = 0 \tag{III.7a}$$

$$V_w \, (0) = -M'_w \, (0) = V(0) = pH + F \tag{III.7b}$$

Aplicando as condições de contorno (III.7a) e (III.7b) determinamos as constantes C_1 e C_2 da expressão (III.5).

Os deslocamentos <u>u</u> podem então ser calculados por

$$u = \frac{1}{j_w} \int_o^z \int_o^z M_w dz \tag{III.8}$$

IV. EXEMPLO

Como exemplo calcularemos os deslocamentos horizontais do painel mostrado na figura (1), sujeito a um carregamento uniformemente distribuído em toda a altura do painel, de valor p = 1,0kN/dm

Temos as dimensões

a = 10 dm, d = 35dm, b = 14dm, e = 30dm, c = 4dm, h = 30dm

A espessura do painel é constante e igual a 2dm. As duas prumadas de vigas horizontais possuem seção transversal 2dm x 5dm.

A estrutura possui 20 andares e o módulo de elasticidade longitudinal do material vale

$E = 2,0 \times 10^5$ kN/dm^2

Para esta estrutura temos

1,

$j_g = j_1 + j_2 = 2{,}0 \times 10^5\ (166{,}67 + 457{,}33)$

$j_g = 1{,}248 \times 10^8$ kN.dm^2

2.

$j_f = EI_f = 2{,}0 \times 10^5 \times 3{,}44846 \times 10^4$

$j_f = 6{,}89692 \times 10^9$ kN.dm^2

3.

Cálculo de s_g

3a) – prumada de vigas com vão 3,5m
Conforme expressão (II.23)

$$s_{g1} = \frac{3x2{,}0x10^5x20{,}833x47^2}{2x30x17{,}5^3}$$

$$s_{g1} = 85.868{,}40kN$$

3b) – pilar de pórtico da direita adjacente ao pilar-parede
Conforme expressão (II.46) com $k_C = 0$ ou Tabela (1).

$$s'_{g2} = 18x2{,}0x\frac{10^5}{30}\left(1+\frac{14}{2x32}\right)\frac{0{,}651}{1{,}7177}$$

$$s'_{g2} = 19.710{,}21kN$$

Conforme expressão (II.45) com k_C= 0 ou Tabela (1) e depois colocando em evidência o termo $\left(1+\frac{c_1}{2\ell_1}\right)$.

$$r_{g2} = 6 \times 2{,}0 \times \frac{10^5}{30} \times 0{,}651 \times \left(1 + \frac{14}{2 \times 32}\right) \times \left[1 + \frac{14}{32} - \frac{0{,}651}{2 \times 1{,}7177}\left(1 + \frac{3 \times 14}{2 \times 32}\right)\right]$$

$r_{g2} = 35660{,}44$

$s_{g2} = r_{g2} + s'_{g2} = 55370{,}65$ kN

A rigidez total à força cortante no painel será

$s_g = s_{g1} + s_{g2} = 1{,}41239.10^5$ kN

A equação diferencial (III.4) em M_w ficará

$-M''_w + 1{,}14605.10^{-3}\, M_w = -p + 1{,}43230.10^{-5}\, M$

e portanto

$\alpha = 3{,}38534 \,.\, 10^{-2}$

$\beta_1 = 1{,}0$

$\beta_2 = 1{,}43230.10^{-5}$

$A = -7{,}16150.10^{-6}$ $B = 8{,}59380.10^{-3}$ $C = -1{,}57814$

As condições de contorno são

$M_w\,(600) = 0;$ $V_w\,(0) = -\,M'_w\,(0) = 600$kN

Para as constantes C_1 e C_2 encontramos

$C_1 = 1{,}29942.10^{-6}$

$C_2 = 17501{,}964$

e para os momentos fletores na parede

$$M_w = 1{,}299942.10^{-6}\ e^{3{,}38534.10^{-2}z} + 17501{,}964\ e^{-3{,}38534.10^{-2}z}$$

$$+6{,}24885.10^{-3}z^2 - 7{,}49863z + 1387{,}94$$

Os deslocamentos horizontais são dados por

$$u = 8{,}01282.10^{-9} \int_o^z \int_o^z M_w dz$$

Os valores encontrados para os deslocamentos horizontais estão apresentados no gráfico da figura (12), onde também são apresentados os resultados obtidos por técnica discreta, utilizando o sistema ANSER[7], desenvolvido no Departamento de Engenharia de Estruturas da EESC-USP.

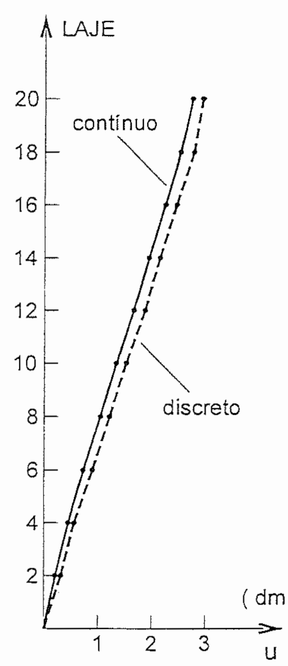


Figura 12 – Deslocamentos horizontais do painel

A diferença máxima observada no topo do painel é de ordem de 8,6%.

Foram também avaliadas as forças normais nos três pilares e os resultados são bastante razoáveis em comparação com os obtidos pelo processo discreto, com algumas imprecisões na base dos dois pilares da direita.

V – CONCLUSÕES

A análise apresentada corrige tratamentos mais antigos que consideram os pilares indeformáveis axialmente. Além dos parâmetros j_g e s_g acrescentamos mais um parâmetro $j_f = EI_f$ onde I_f é o momento de inércia do conjunto das áreas das seções transversais dos elementos verticais em relação ao seu centróide não considerando o momento de inércia das seções em relação aos seus próprios centróides. Chegamos a este parâmetro supondo que a seção plana horizontal do painel permaneça plana.

Conforme vimos, com um exemplo, os deslocamentos horizontais obtidos são bastante próximos dos considerados "exatos", sendo pequeno o esforço de cálculo realizado.

Avaliamos também os esforços normais nos pilares, que apesar de alguma imprecisão na base dos mesmos, são também bem razoáveis quando comparados com os obtidos com o processo discreto.

Mais importante ainda é constatar que podemos substituir o complexo painel geral pela associação de uma parede e um pórtico para barras bi-articuladas com rigidez praticamente equivalente. Este resultado é bastante útil na avaliação da distribuição das forças horizontais em uma estrutura de edifício alto considerada como uma associação tridimensional de painéis quaisquer. Neste caso cada painel da edificação seria substituído por uma parede e um pórtico associados, no plano do painel original, e a estrutura tridimensional equivalente seria composta somente de paredes e pórticos. Como tanto a parede como o pórtico pode ser considerado painel deformável por momento fletor e força cortante, com seus respectivos parâmetros, a estrutura equivalente pode ser tratada como uma associação tridimensional de painéis deformáveis por momento fletor e força cortante conforme vimos em MANCINI e SAVASSI[3].

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